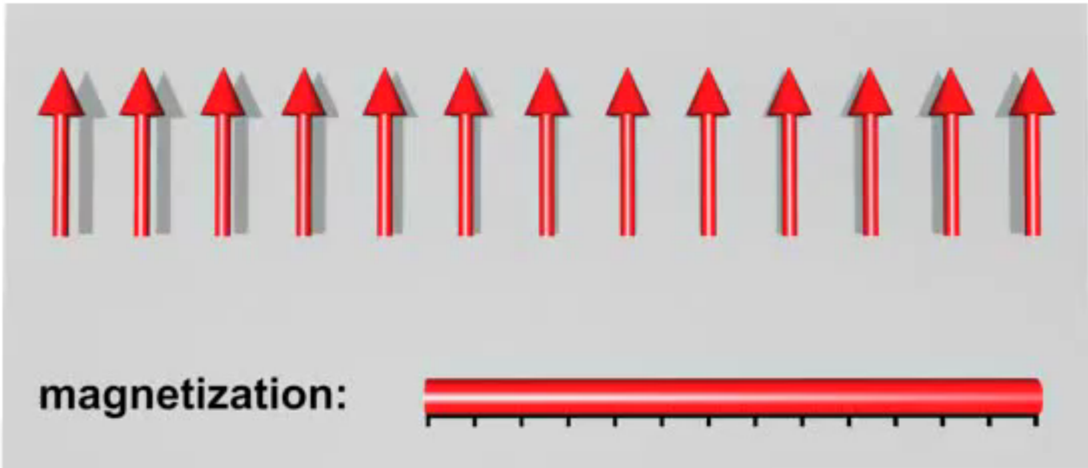
Magnetization Dynamics I:

Fundamentals of spin dynamics and Brillouin light scattering

Burkard Hillebrands

Fachbereich Physik and Landesforschungszentrum OPTIMAS,
Technische Universität Kaiserslautern, Germany

Ferromagnetic spin chain: magnon



Animation: H. Schultheiss

Lecture 1 - CONTENT

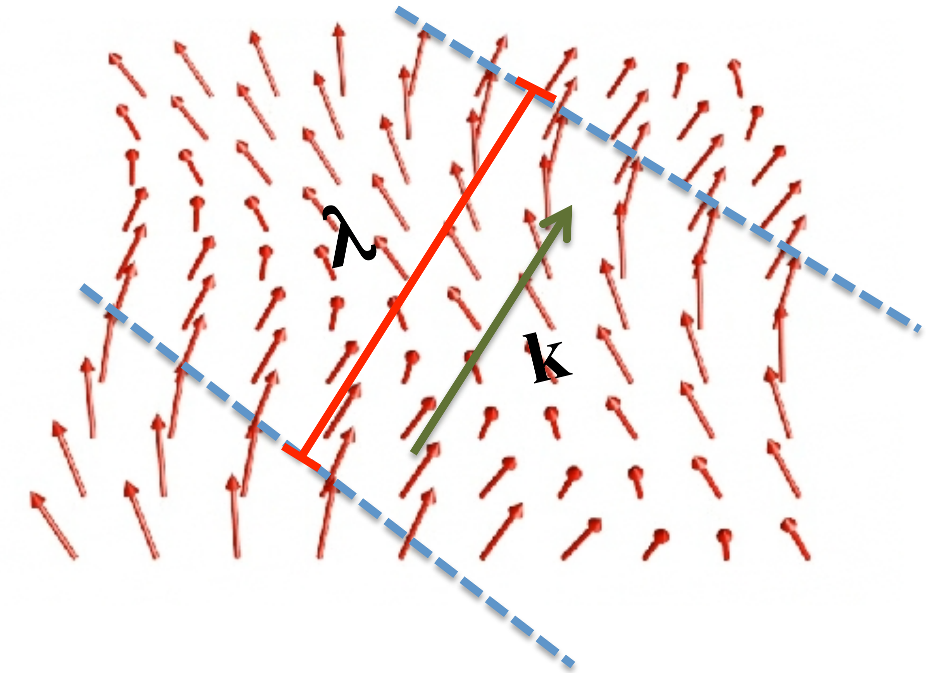
- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

Lecture 1 - CONTENT

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

Coherent dynamics: spin waves

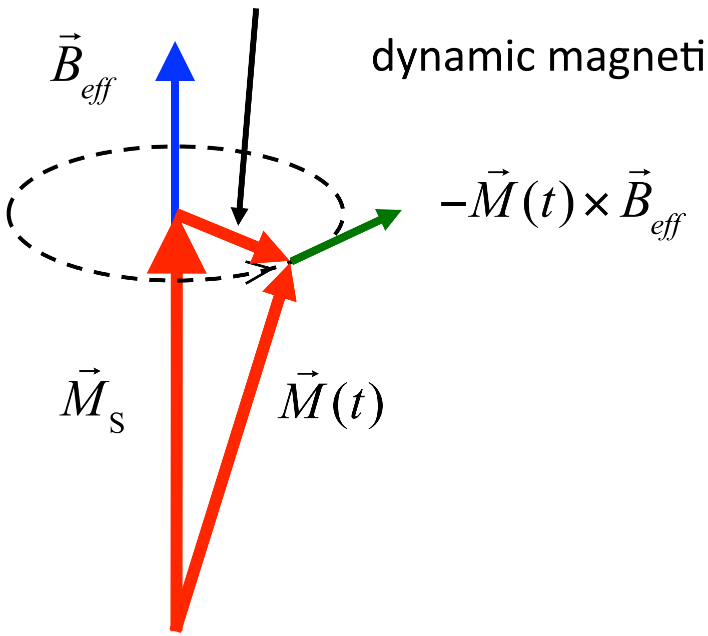
Spin wave: collective motion of magnetic moments



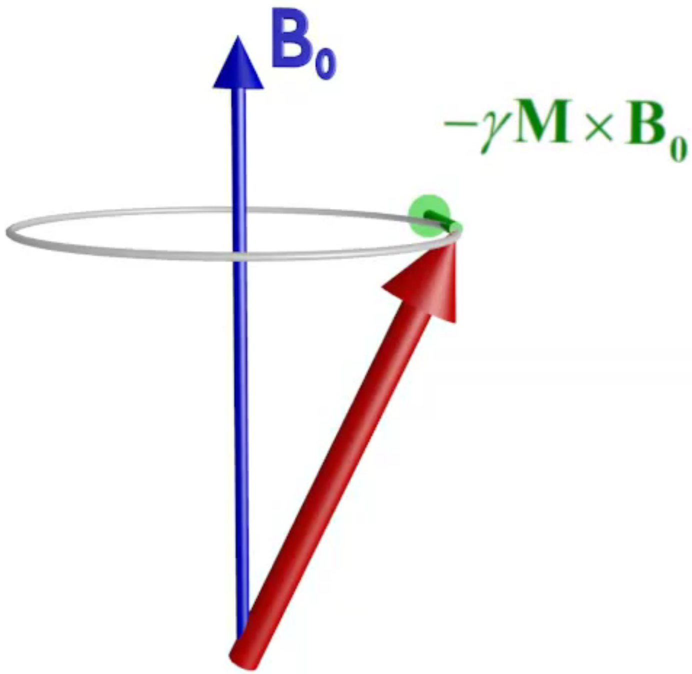
Landau-Lifshitz torque equation

$$\frac{1}{|\gamma|}\frac{d\vec{M}(t)}{dt} = -\vec{M}(t)\times\vec{B}_{eff}(t)$$

$$\vec{m}(\vec{r},t)=\vec{m}_0(\vec{r})\times e^{i(\vec{k}\vec{r}-\omega t)}$$



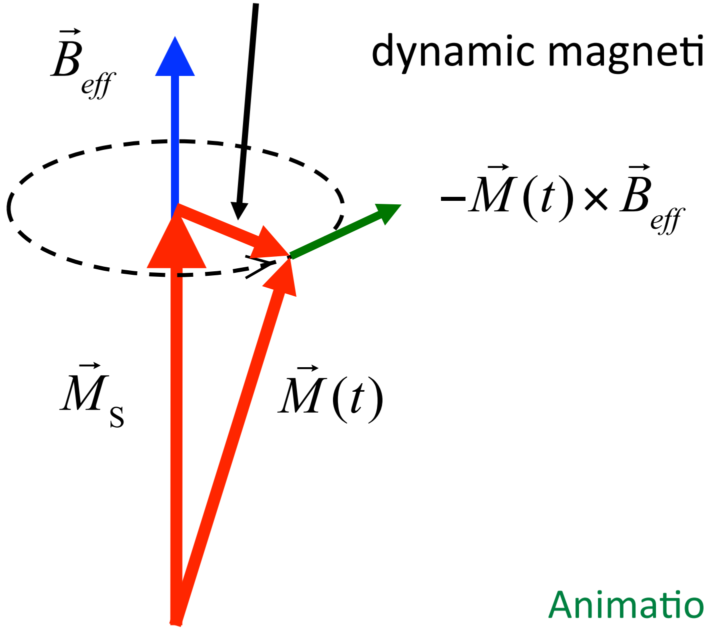
Coherent dynamics: spin-wave decay



Landau-Lifshitz torque equation

$$\frac{1}{|\gamma|}\frac{d\vec{M}(t)}{dt} = -\vec{M}(t) \times \vec{B}_{eff}(t) + \frac{\alpha}{M_s}\vec{M}(t) \times \frac{d\vec{M}(t)}{dt}$$

$$\vec{m}(\vec{r},t) = \vec{m}_0(\vec{r}) \times e^{i(\vec{k}\vec{r} - \omega t)}$$

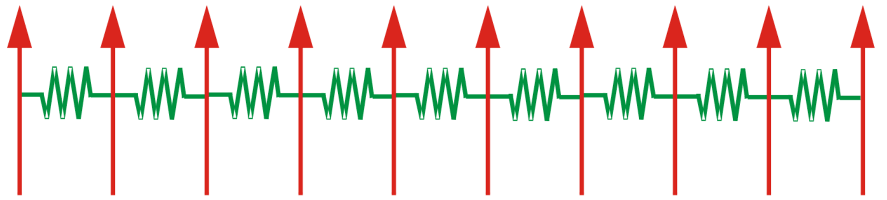


Animation: D.A. Bozhko

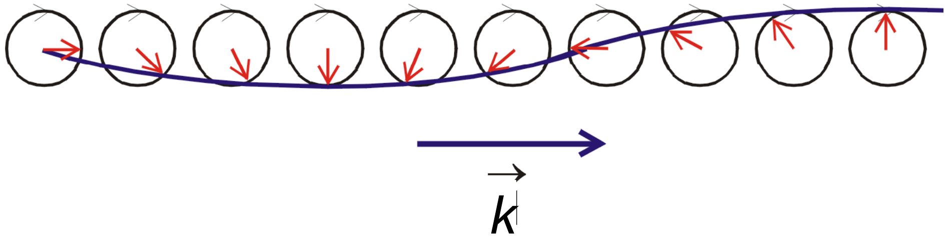
Spin waves

Two types of energy contributions

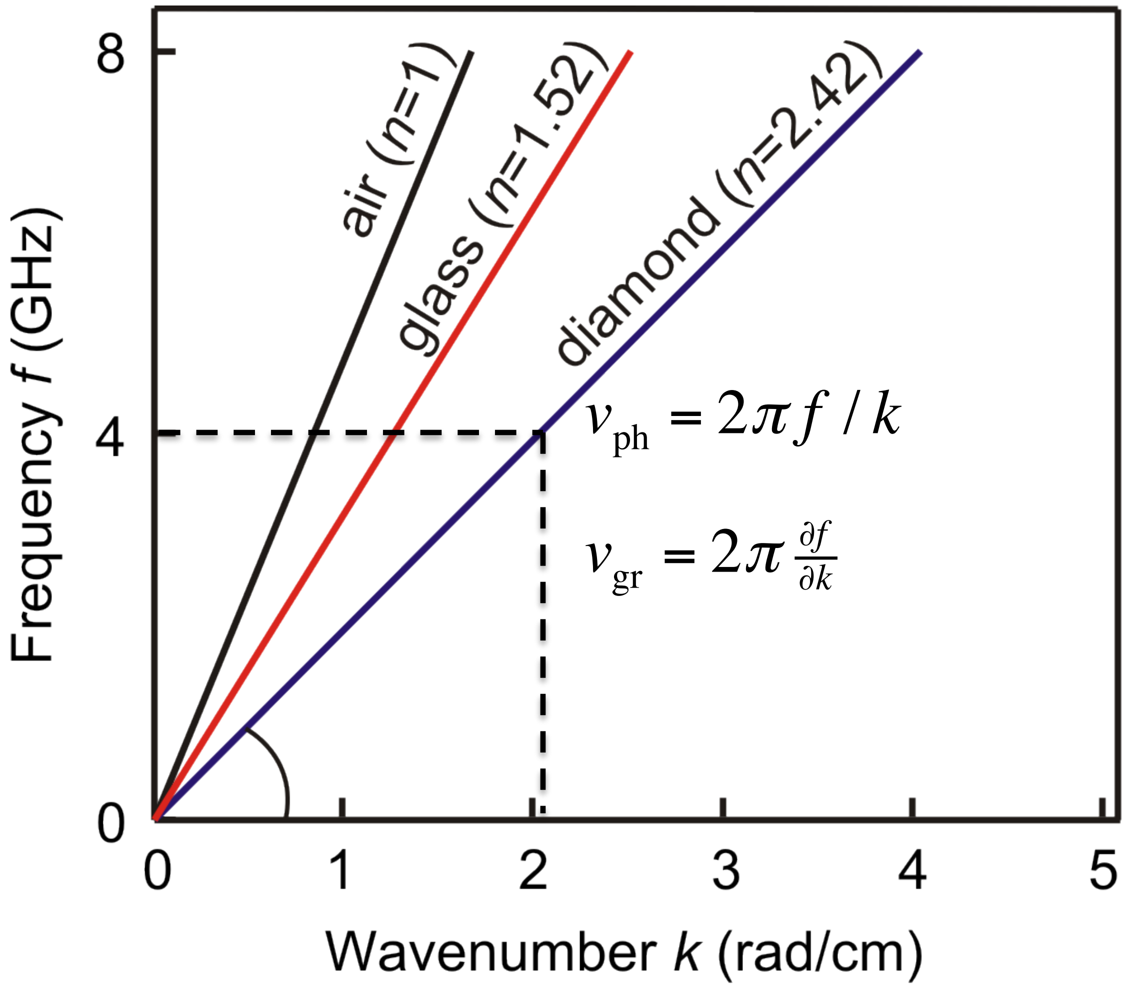
- exchange energy:
  generated by twist of neighbored spins



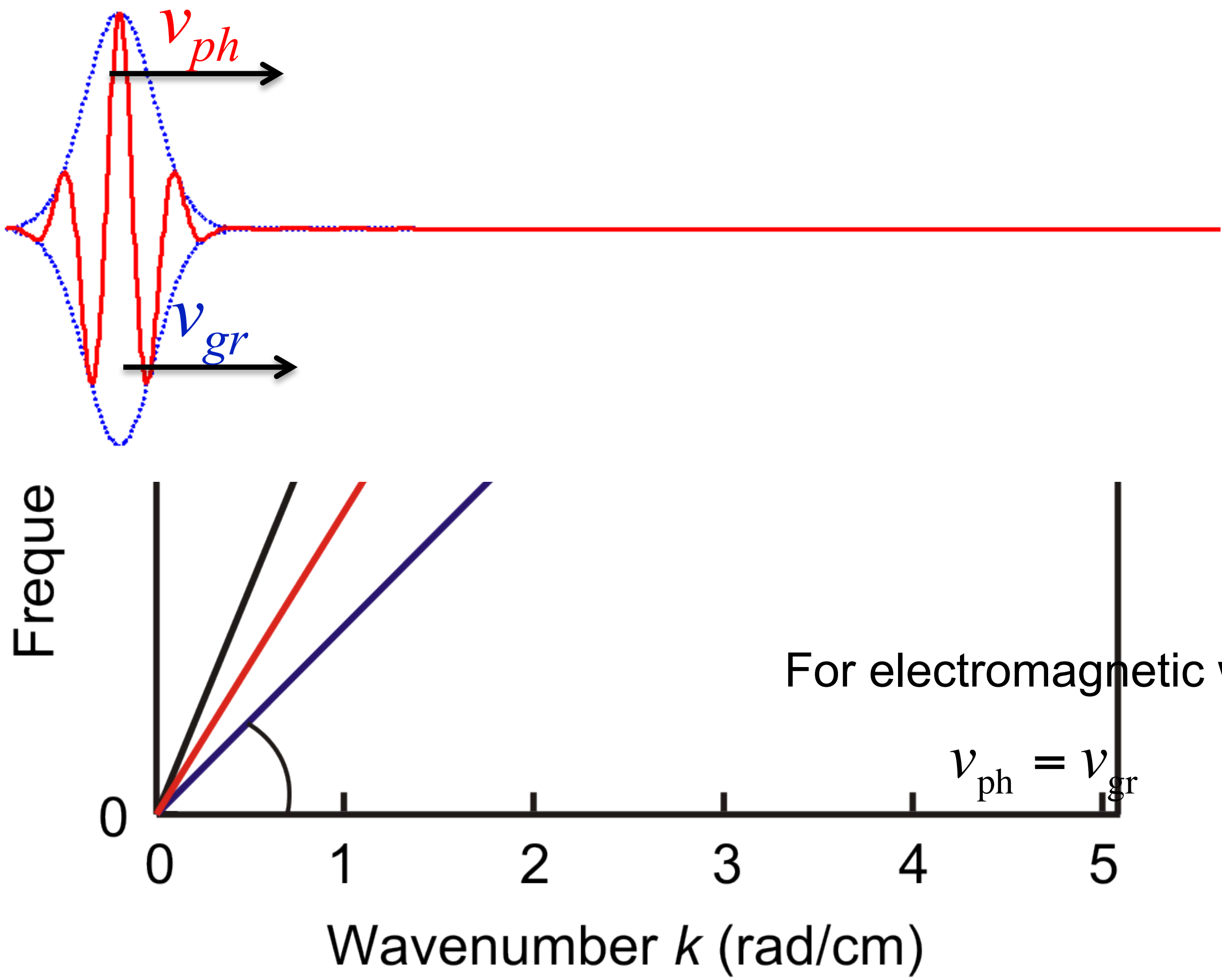
- dipolar energy:
  generated by magnetic poles in long-wavelength spin waves



Dispersion of electromagnetic wave



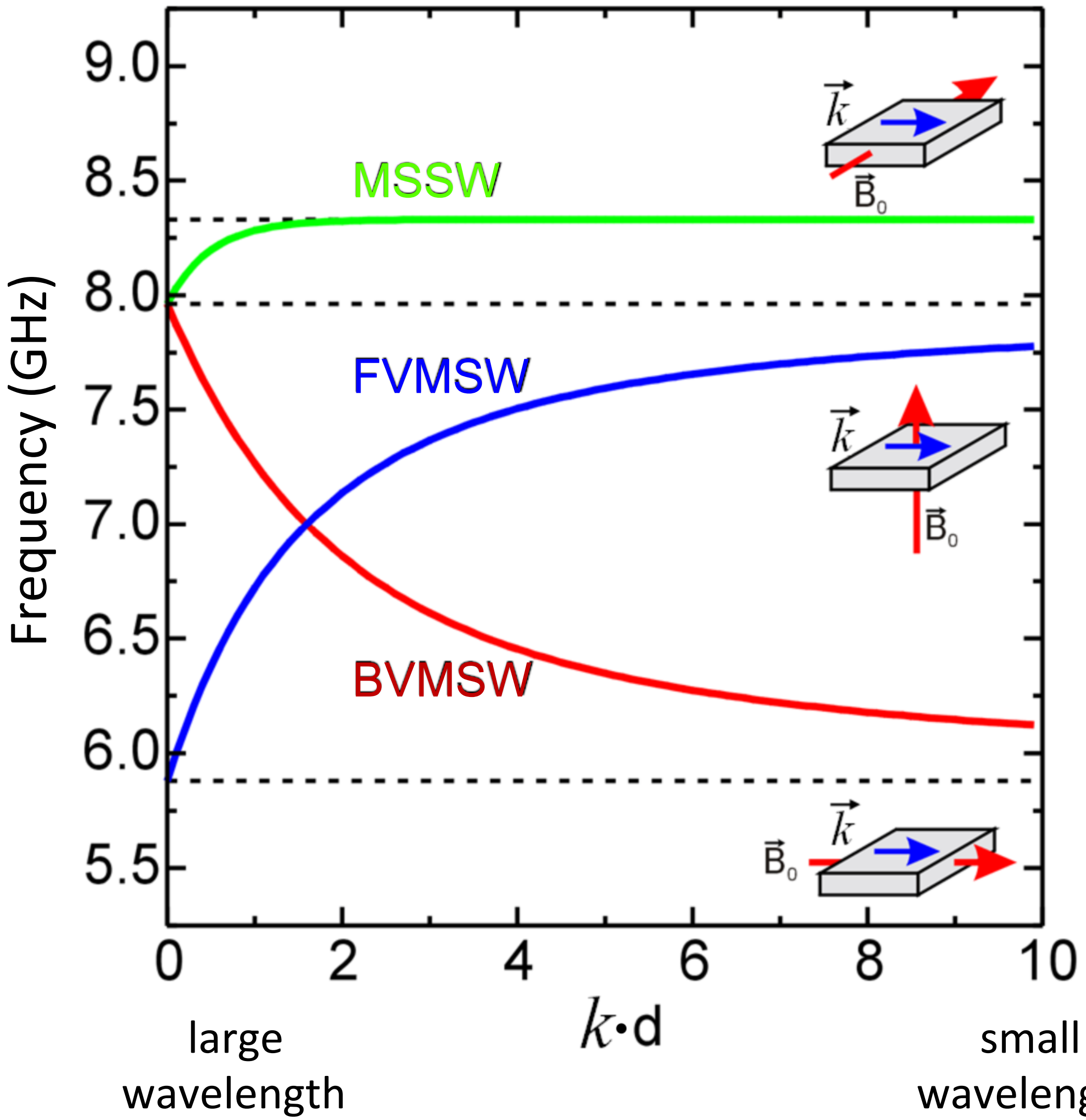
Dispersion of electromagnetic wave



For electromagnetic wave:

$v_{\mathrm{ph}} = v_{\mathrm{gr}}$

Dipolar spin waves

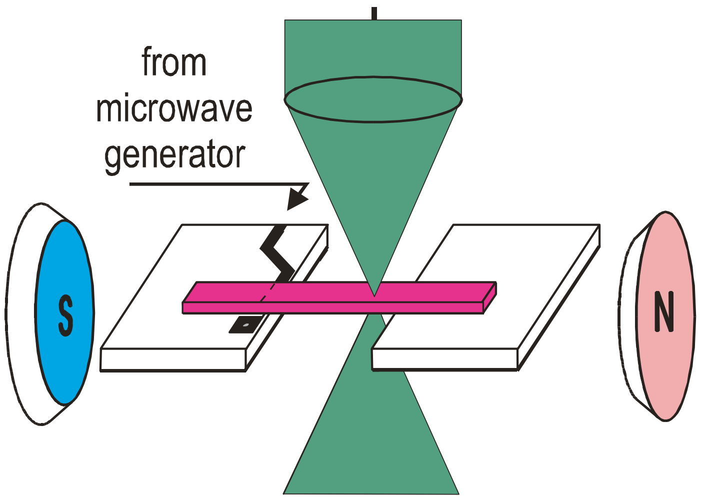


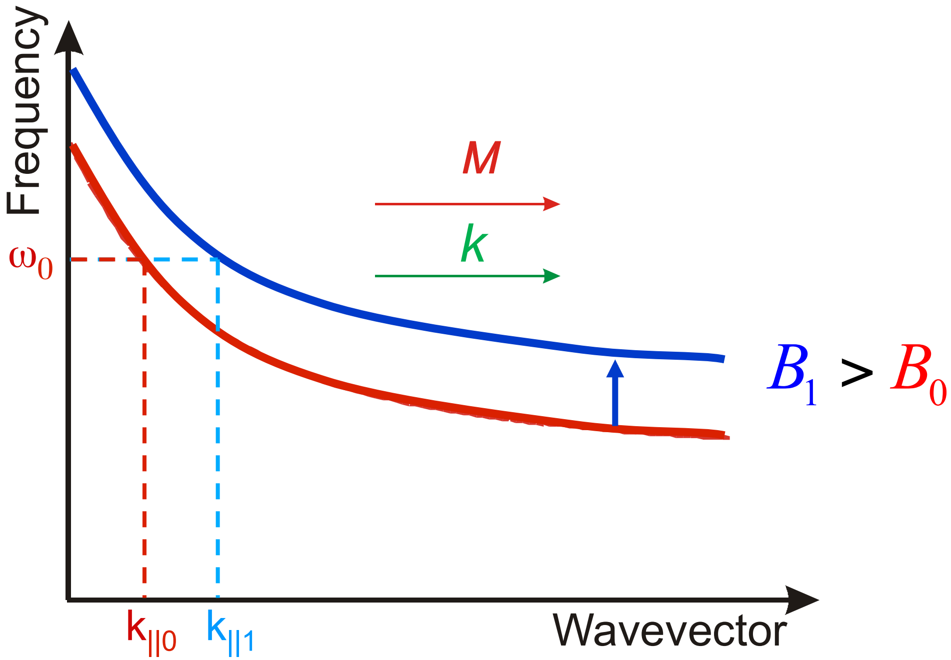
Magnetostatic Surface Spin Waves

Forward Volume Magnetostatic Spin Waves

Backward Volume Magnetostatic Spin Waves

Control of spin wave propagation



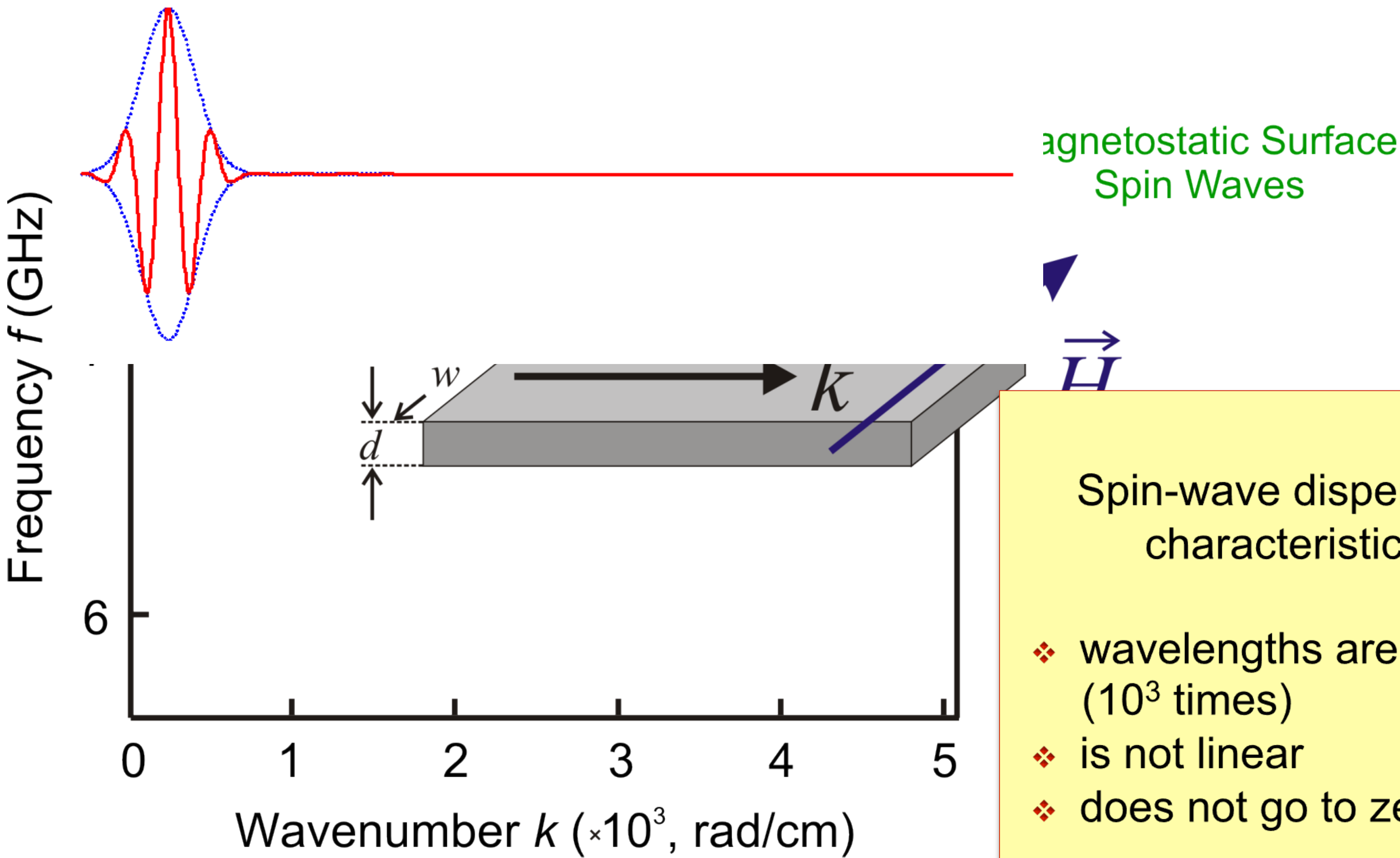


Wavevector **k**:

$k_{parallel}$ defined by input frequency and dispersion

Dispersion shifted vertically by change in magnetic field

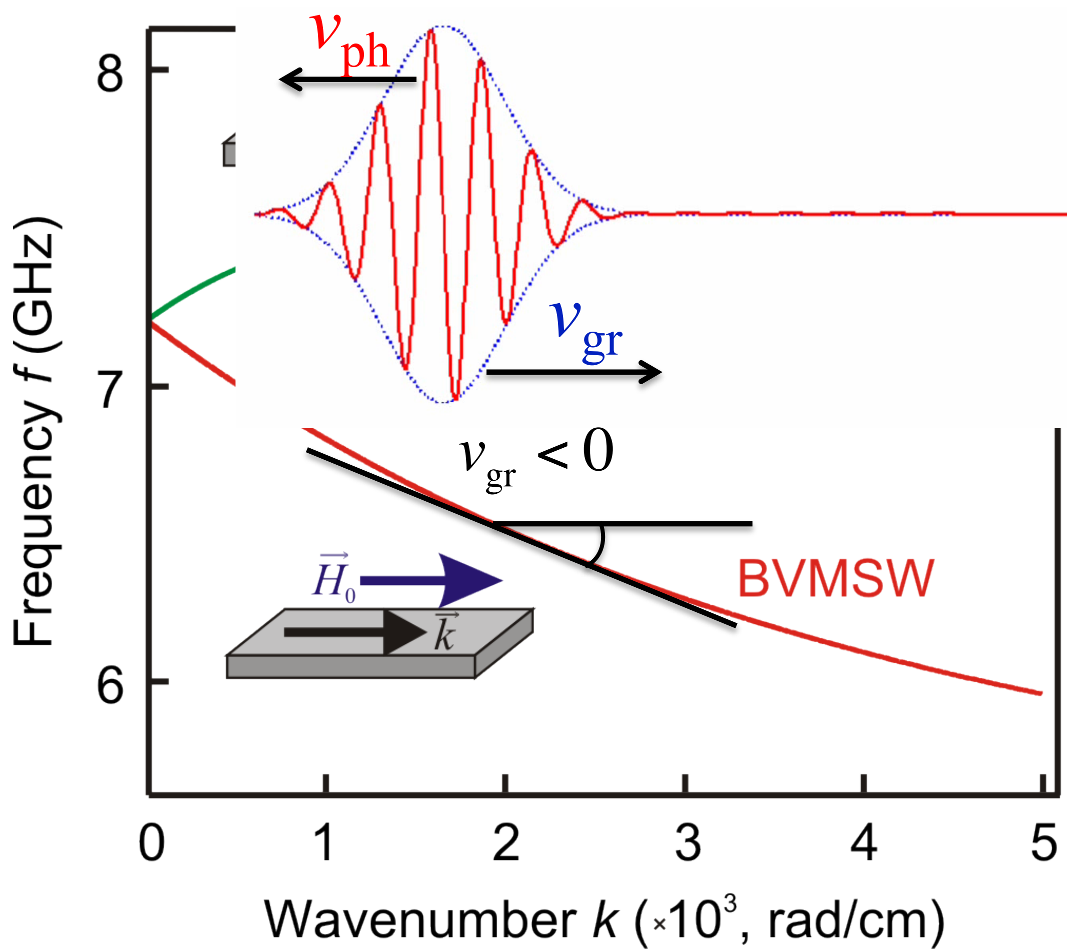
Dispersion curves for spin waves



Spin-wave dispersion characteristics:

- wavelengths are smaller (10^3 times)
- is not linear
- does not go to zero

Dispersion curves for spin waves



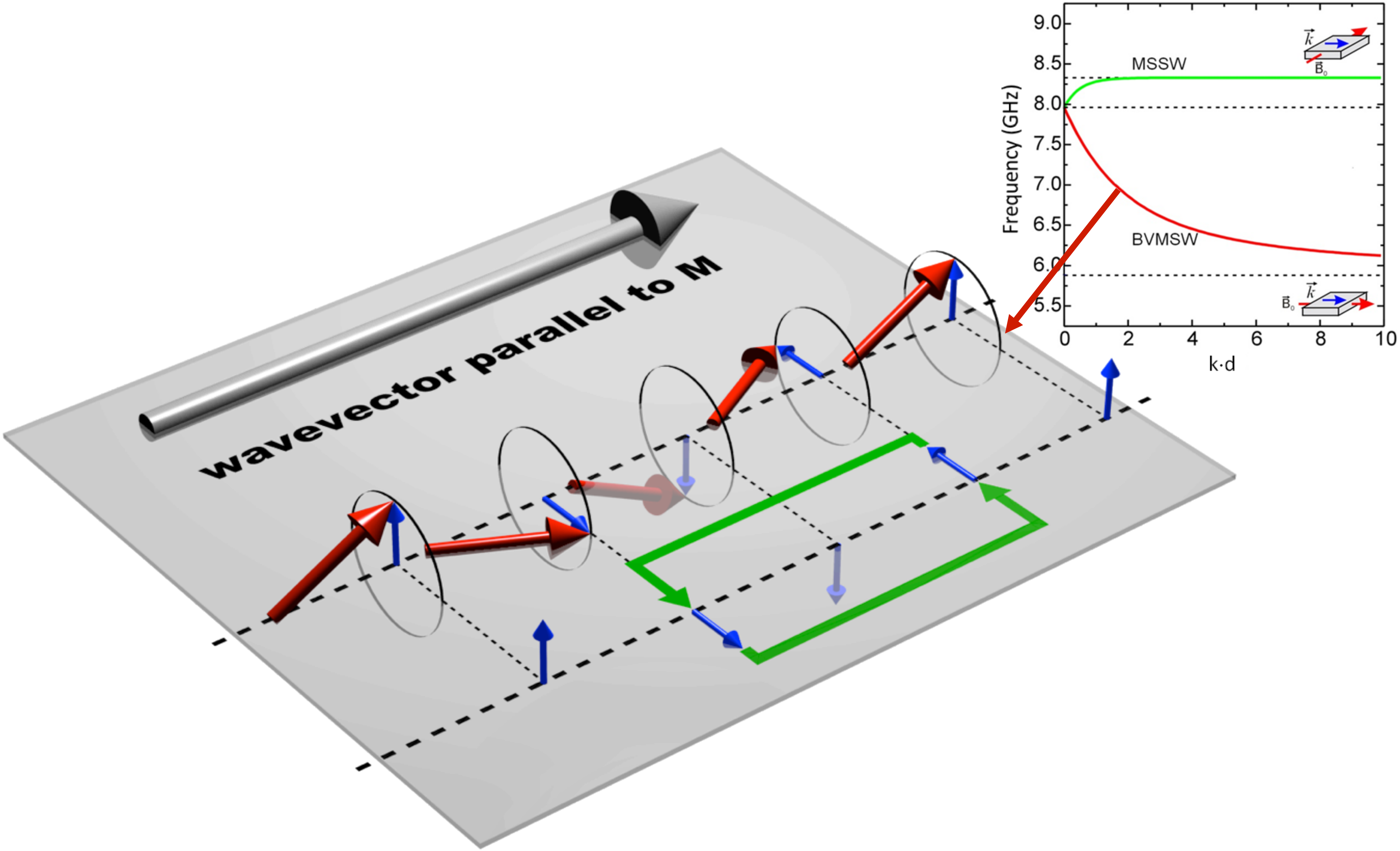
For BVMSWs:

$$v_{gr} < 0, \quad v_{ph} > 0$$

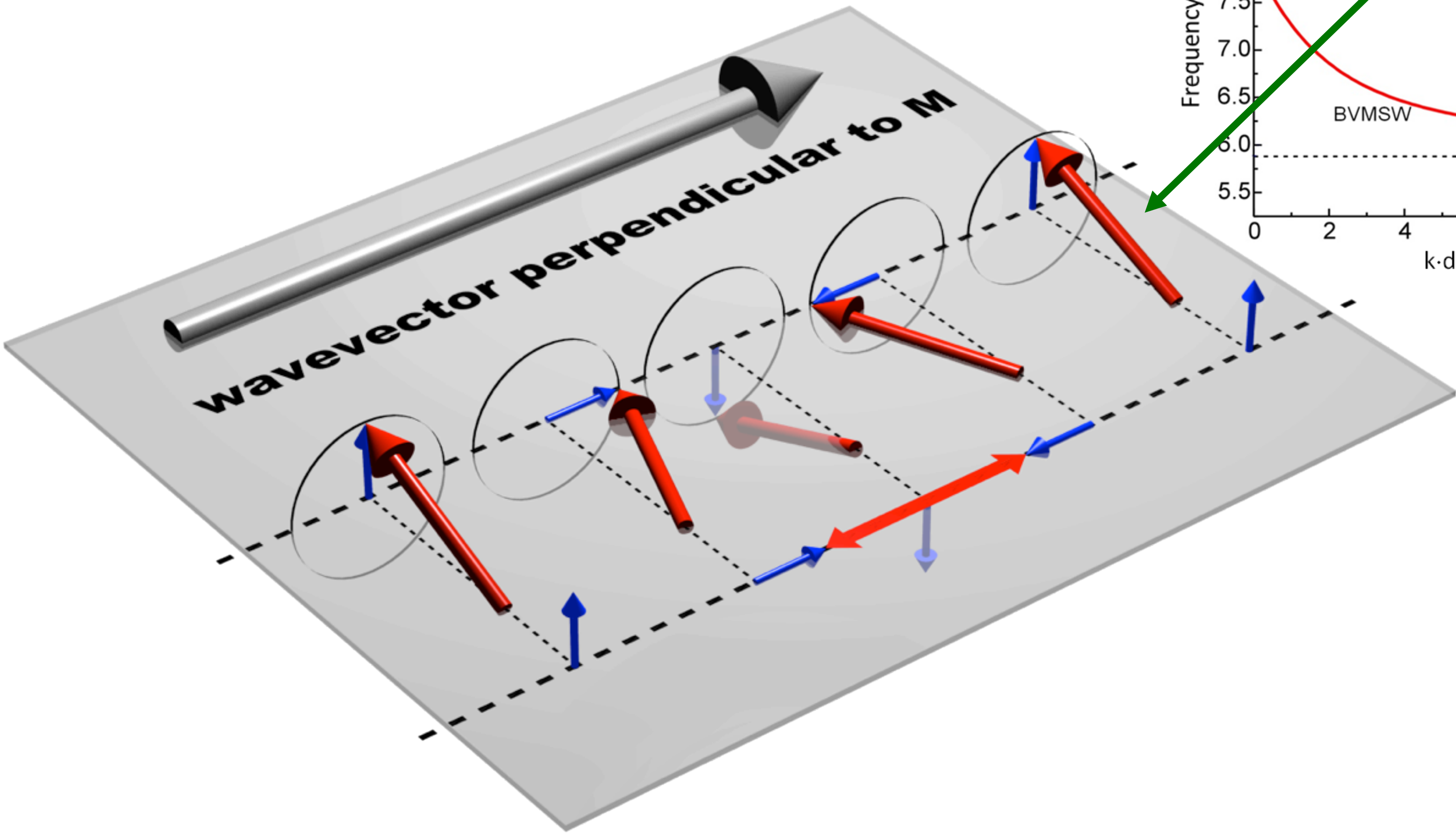
group and phase velocities have different signs

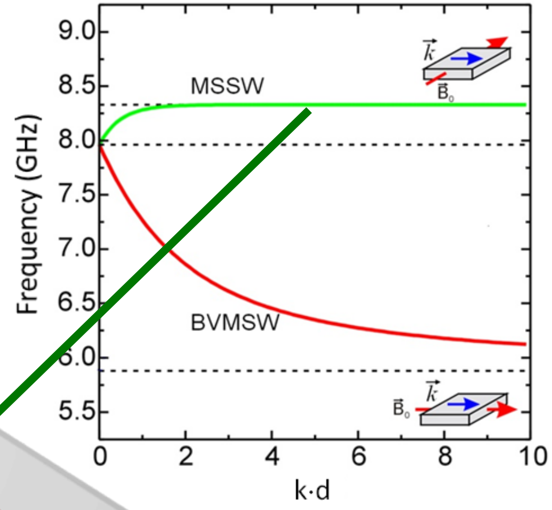
TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Backward volume magnetostatic spin wave



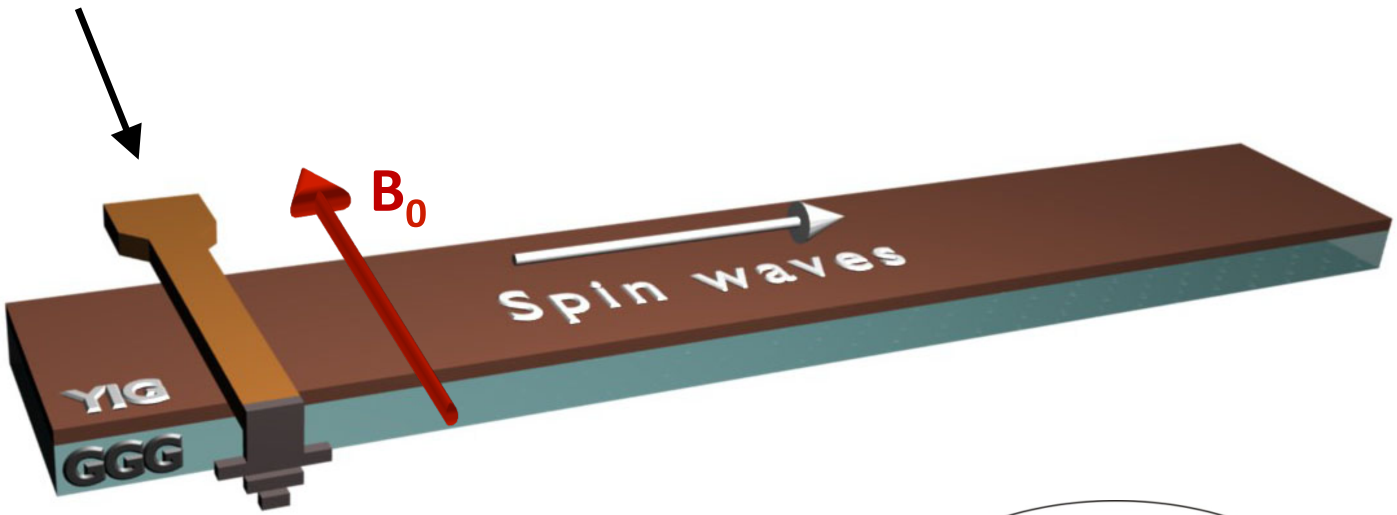
Magnetostatic surface spin wave

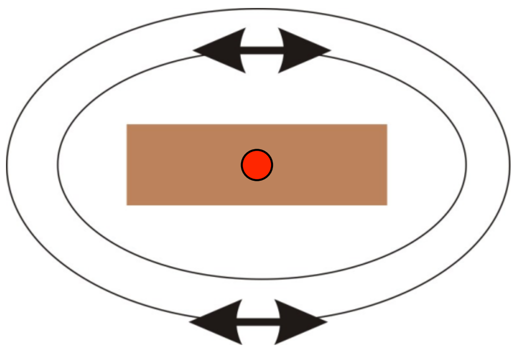




Excitation of dipolar spin waves

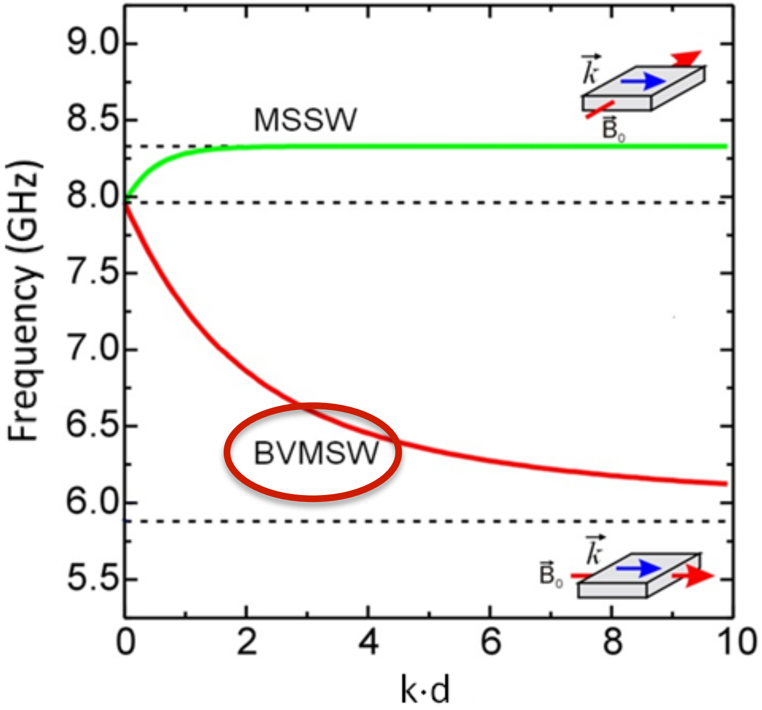
Input microwave signal



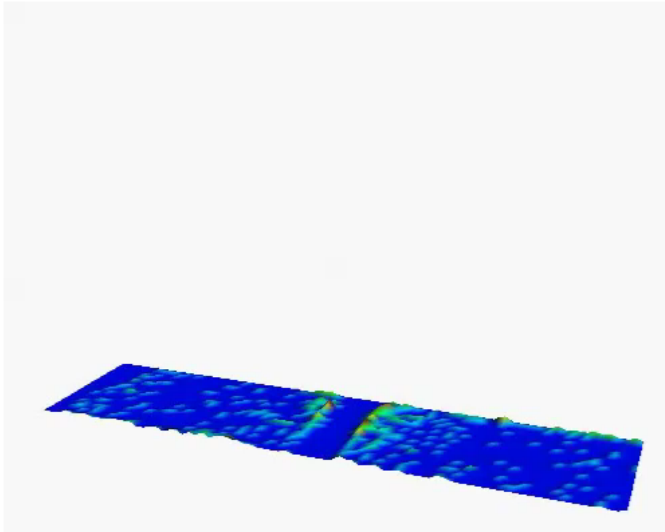


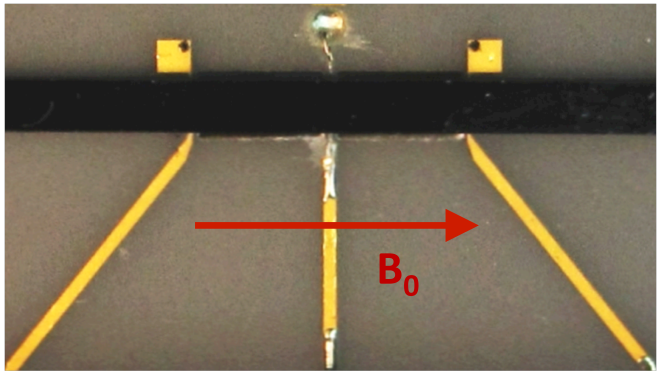
Alternating magnetic field

Backward volume magnetostatic spin waves (BVMSW)



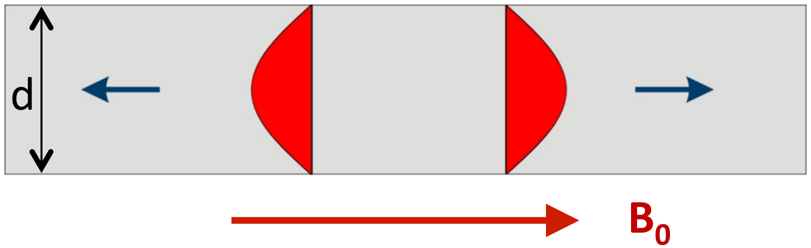
Excitation of BVMSW
measured with
Brillouin light scattering microscopy



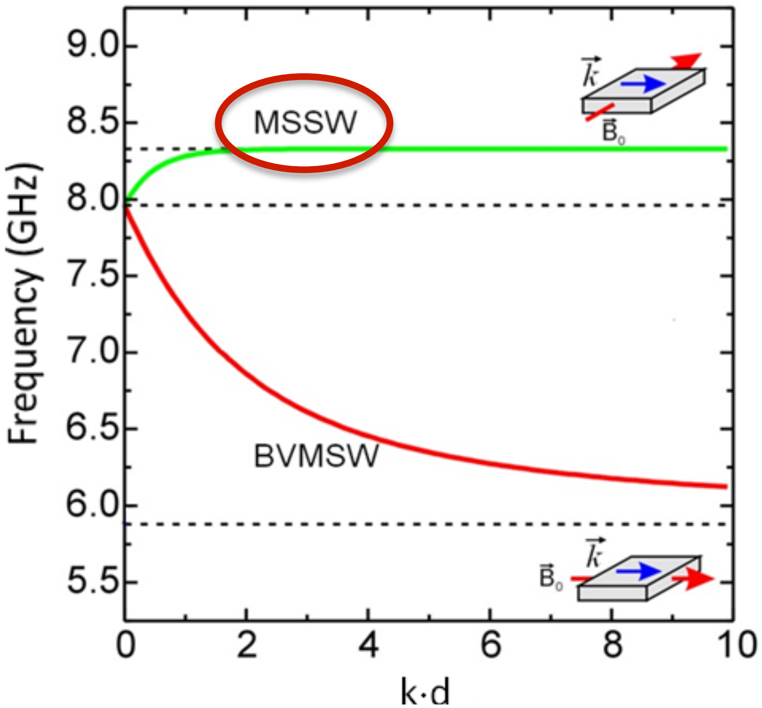


Dynamic magnetization profile

$$m_x \sim \cos(kx)$$

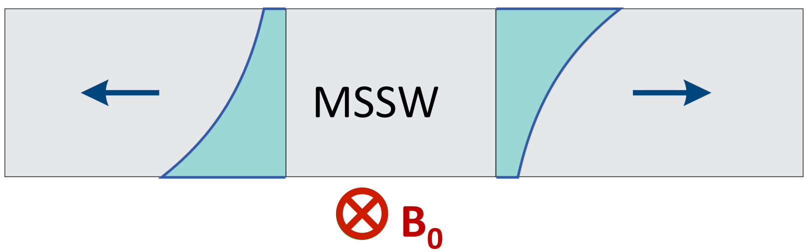


Magnetostatic surface spin waves (MSSW)

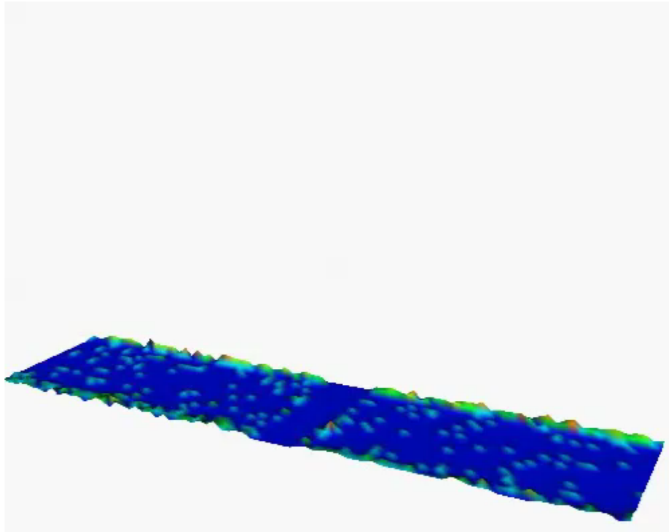


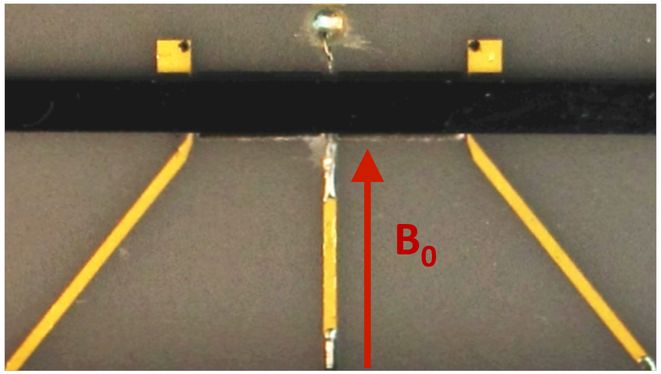
Dynamic magnetization profile

$$m_x \sim \exp(-kx)$$

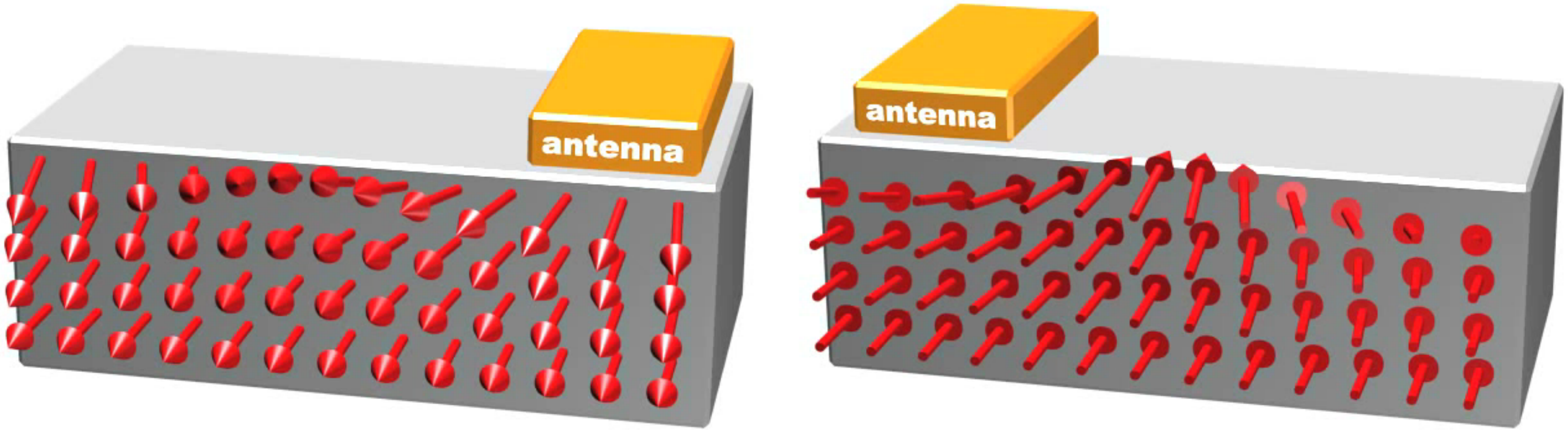


Excitation of MSSW
measured with
Brillouin light scattering microscopy





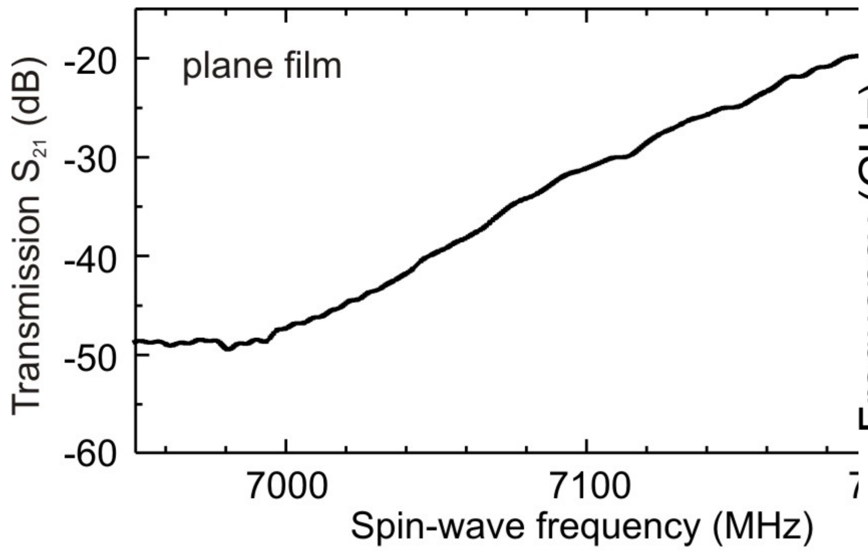
Magnetostatic surface spin wave

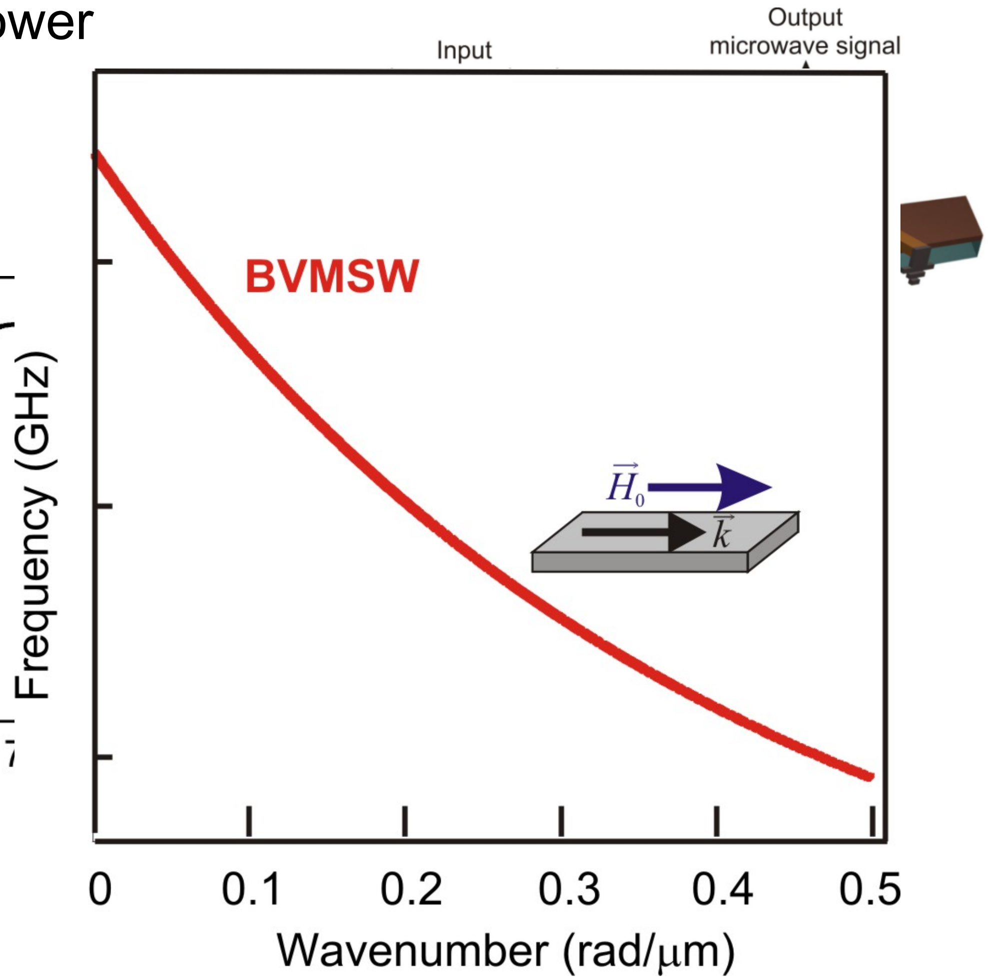


Animation: H. Schultheiss

BVMSW transmission characteristics

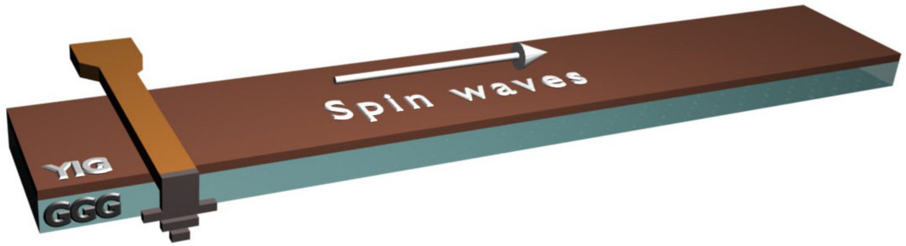
Dependence of the transmitted power on frequency





Chumak, et al., APL **93**, 022508 (2008)

Spin-wave waveguide



Frequency: $$f(k) = \gamma \sqrt{H_0 + 4\pi M_0 \frac{1 - \exp\left\{-\sqrt{(n\pi / w)^2 + k^2}\, d\right\}}{\sqrt{(n\pi / w)^2 + k^2}\, d}}$$

H_0 – magnetic field

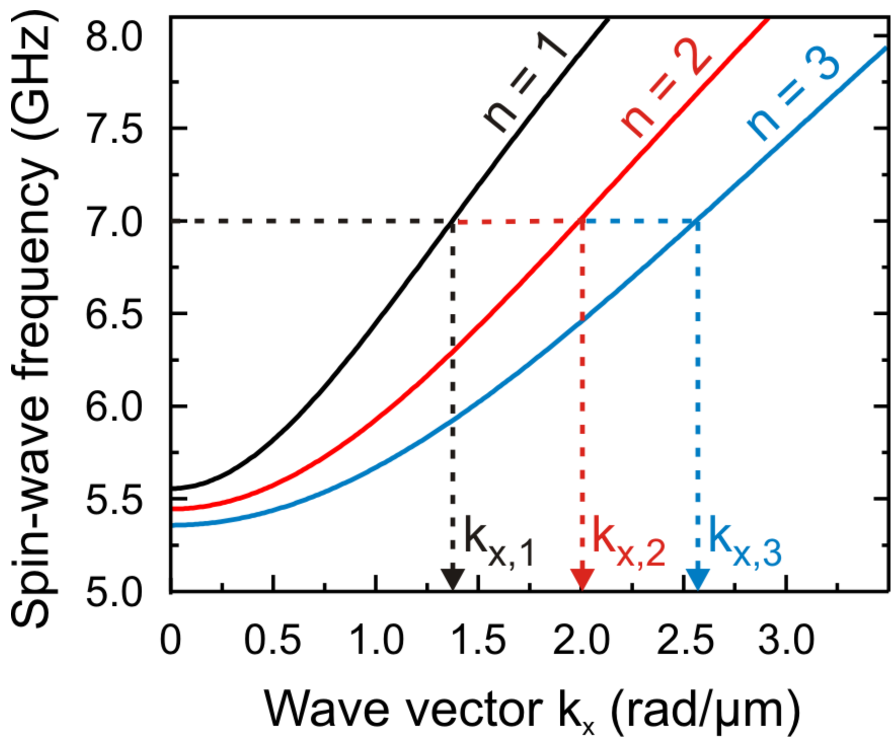
M_0 – saturation magnetization

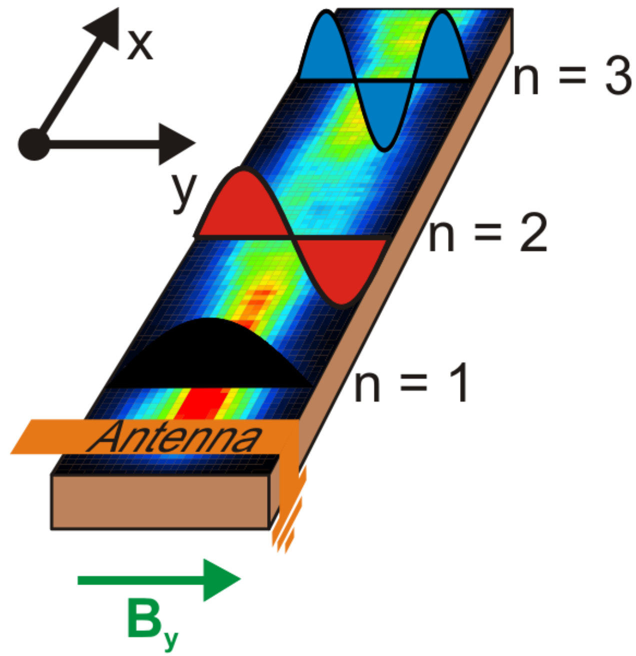
d – film thickness

w – waveguide width

n – transverse mode order

$Ni_{81}Fe_{19}$ waveguide





Material: $Ni_{81}Fe_{19}$

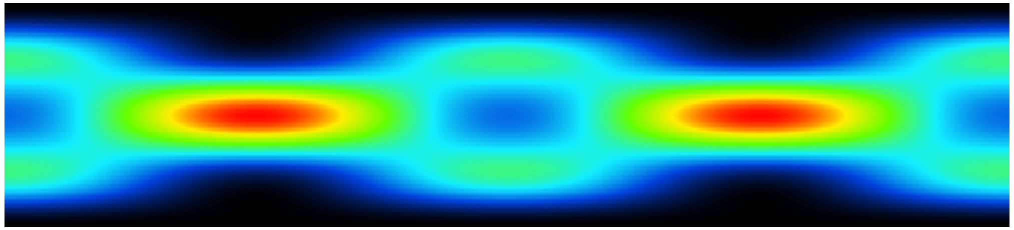
k_x: propagating spin wave

k_y: lateral standing spin wave with mode order n

Wave superposition

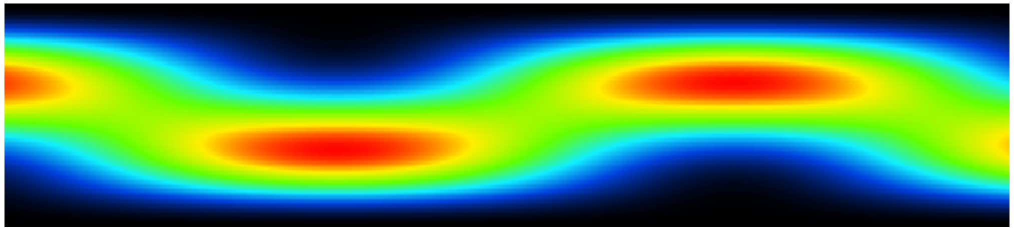
$$I(x,y)=\left|A_1 e^{-i(k_1 x)}\cos(n_1\frac{\pi}{w}y)+A_2 e^{-i(k_2 x)}\cos(n_2\frac{\pi}{w}y)+\ldots\right|^2$$

Modes
n=1 & 3

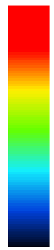


[1,2]

n=1 & 2



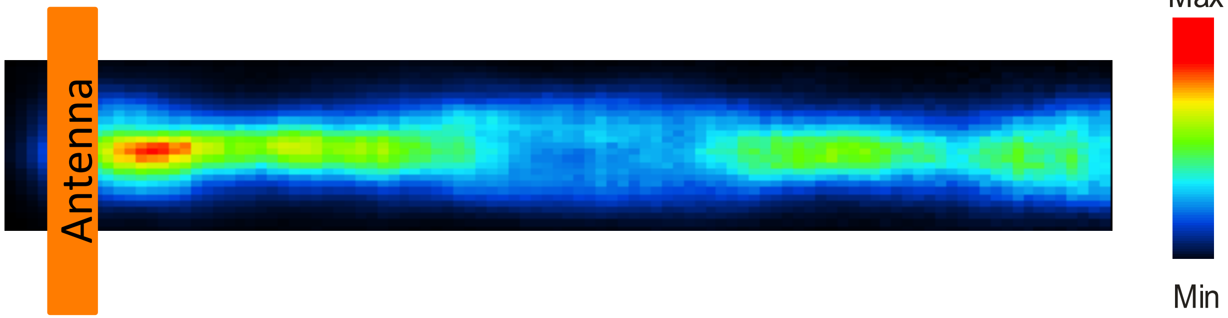
Max



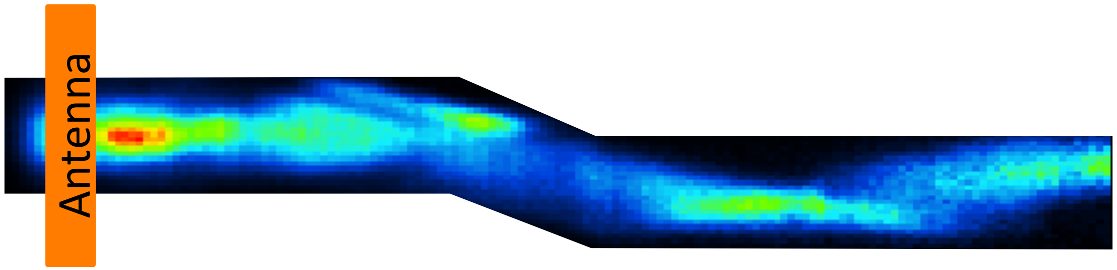
Min

Influence of a skew

Reference waveguide

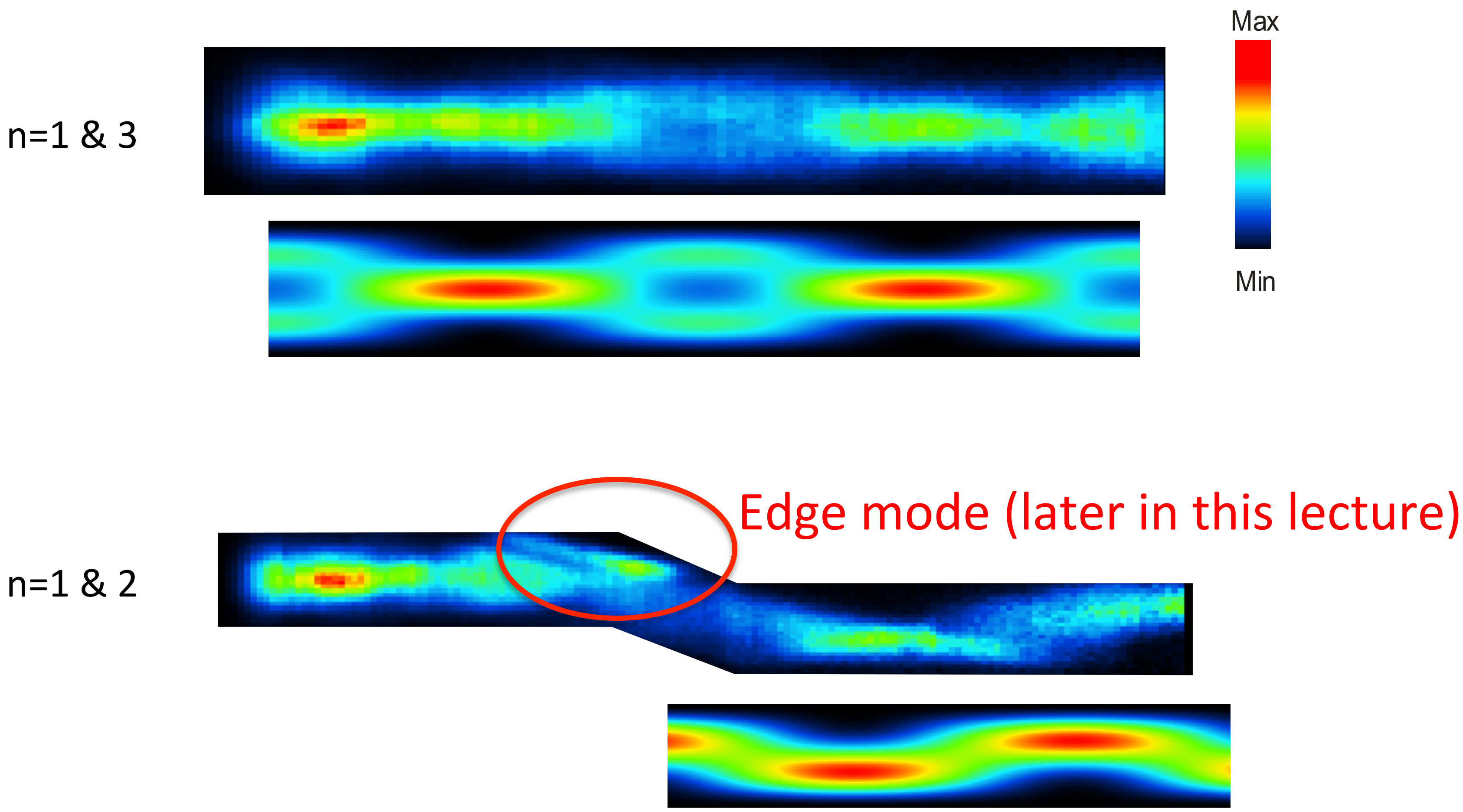


Waveguide with skew (1 µm)



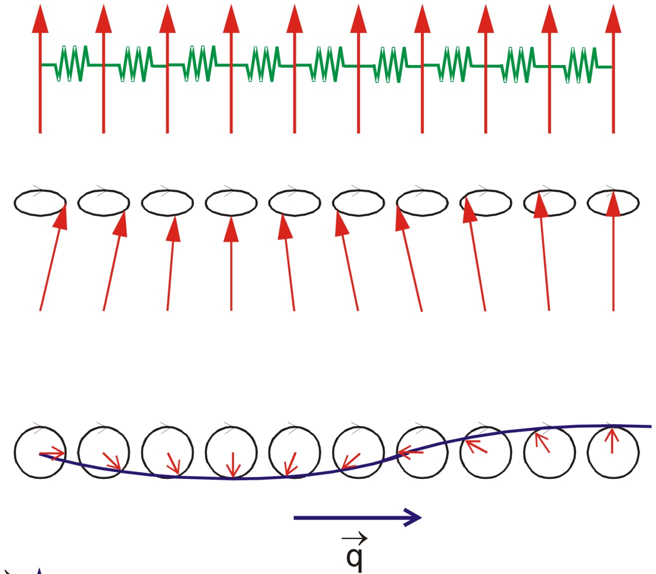
- Changing interference patterns (n=1&3 to n=1&2)
- Edge mode: asymmetric source

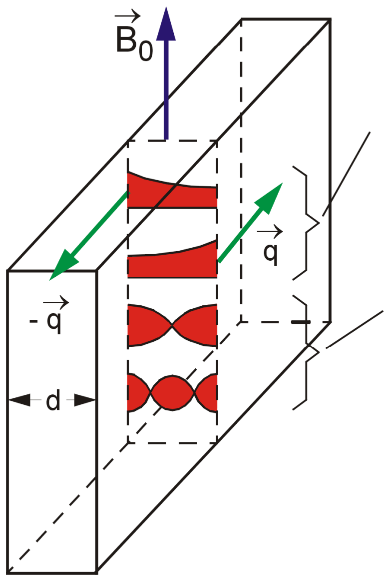
Supporting numerical simulations



Excitation of the second width mode

Spin waves in a thin magnetic film





Dipolar Damon-Eshbach modes:

$$\frac{\omega}{\gamma}=B_0(B_0+\mu_0 M_S)+\left(\frac{\mu_0 M_S}{2}\right)^2\left(1-e^{-2q_{\|}d}\right)$$

Standing spin waves:

$$\frac{\omega}{\gamma}=\frac{2J_{ex}}{M_S}\cdot q^2=\frac{2J_{ex}}{M_S}\cdot\left(\frac{n\pi}{d}\right)^2 \quad , n=\pm1,\pm2,\pm3,\ldots$$

J_{ex} : exchange constant

Exchange modes

Small wavelength: exchange interaction

$$\vec{B}_{\text{exch}} = \frac{2J_{\text{ex}}}{M_s^2}\vec{\nabla}^2\vec{M} = \frac{D}{M_s}\vec{\nabla}^2\vec{M}$$

must be considered.

Resonance condition for wavevector component perpendicular to film:

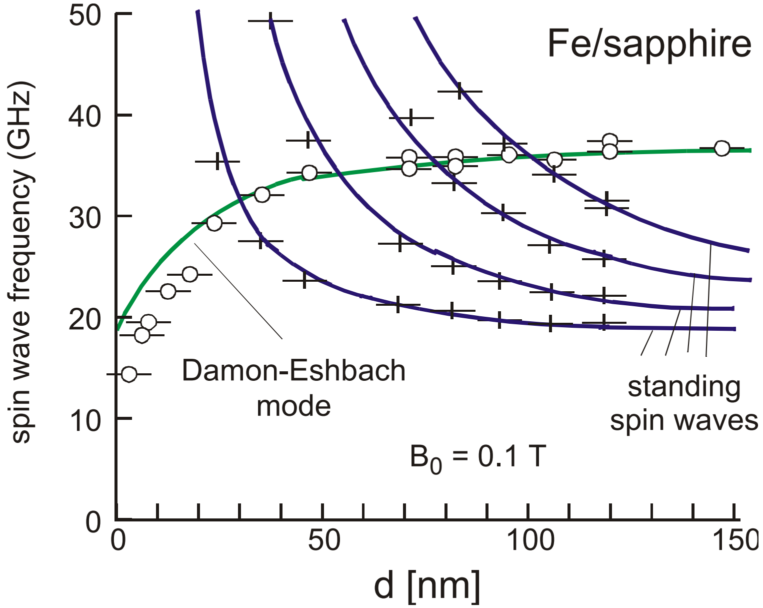
$$q_\perp = n\frac{\pi}{d}; \quad n = \pm 1, \pm 2, \pm 3 \ldots$$

approximative calculation of exchange modes:

(outside crossing regimes with Damon-Eshbach modes)

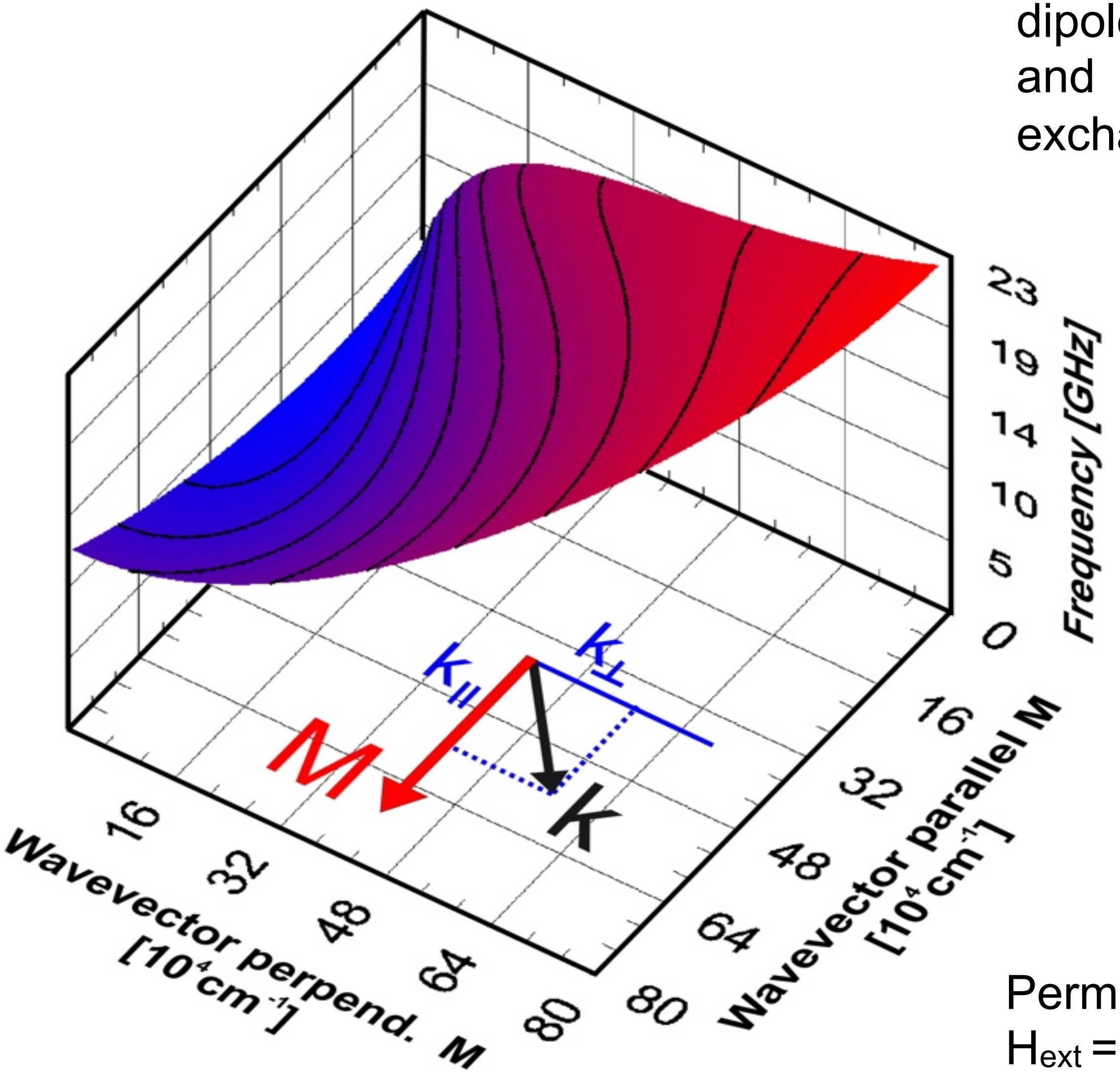
$$\left(\frac{\omega}{\gamma}\right)^2 = \left(B_0 + Dq^2\right)\left(B_0 + \mu_0 M_s + Dq^2\right)$$

with: $D = 2J_{\text{ex}} / M_s$



P. Grünberg et al., JMMM **28**, 319 (1982)

Propagation at oblique in-plane angle



dipole-dipole interaction
and
exchange interaction

Permalloy film (15nm)
H_{ext} = 500 Oe

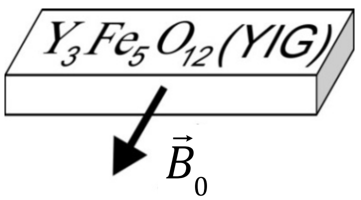
Dipolar and exchange spin waves

Landau-Lifshitz equation: $\frac{\partial \vec{M}}{\partial t} = -|\gamma| \vec{M} \times B_{\text{eff}}$

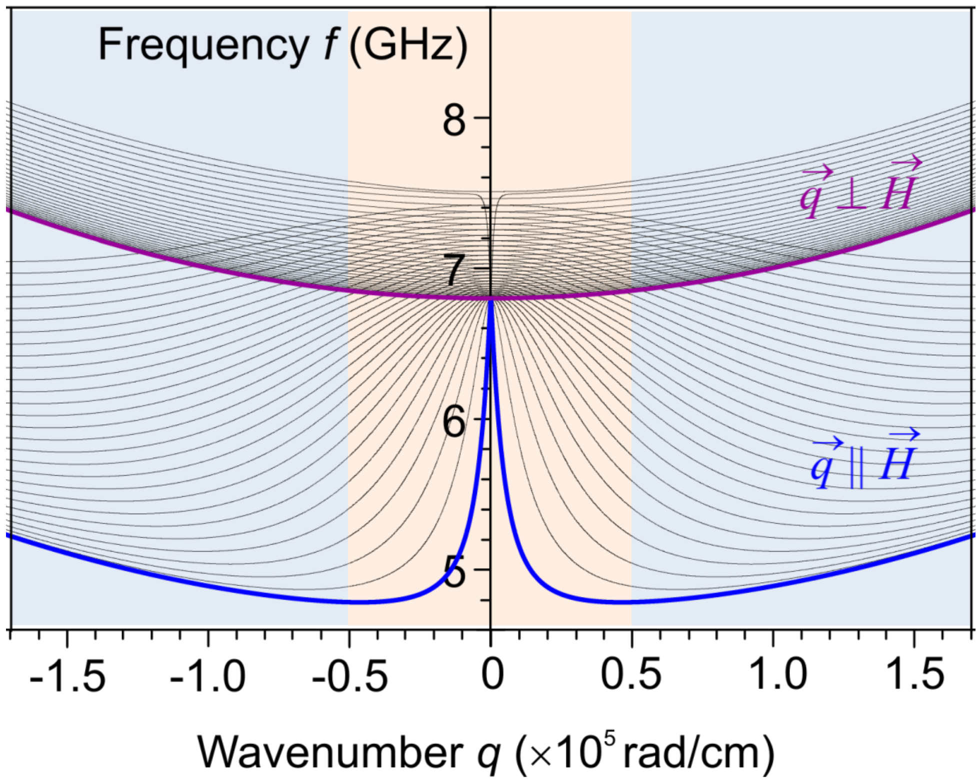
$$\vec{B}_{\text{eff}}(\vec{r}) = \vec{B}_0 + \mu_0 \int_V \tilde{G}(\vec{r},\vec{r}\,')\vec{M}(\vec{r}\,')dr'^3 + \frac{J_{\text{ex}}}{\gamma M_s}\vec{\nabla}^2\vec{M} + \ldots$$

dipolar interaction

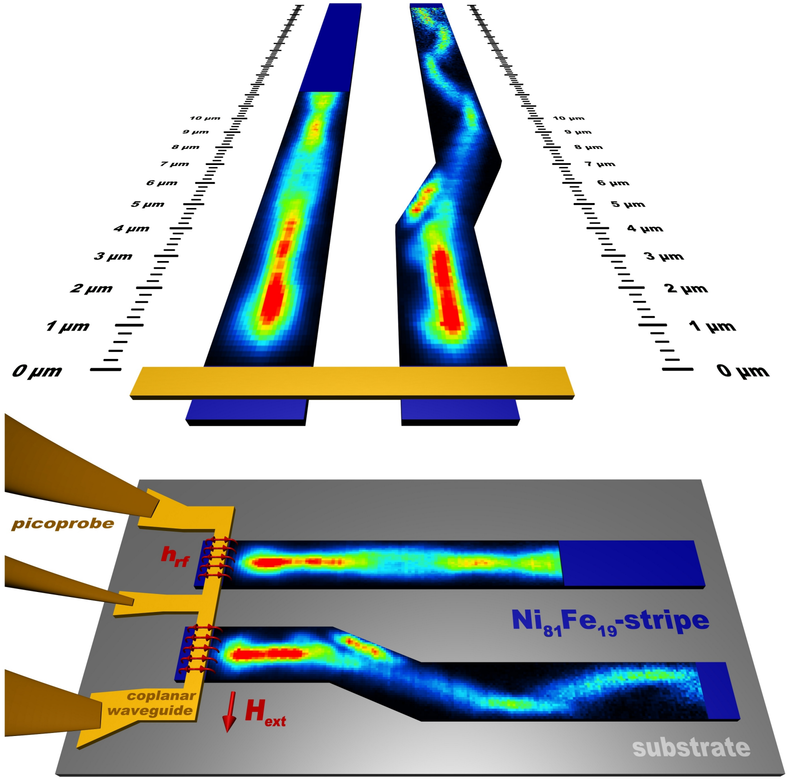
exchange interaction



$B_0/\mu_0 = 1710$ Oe



Spin wave bus



Travelling magnons allow one to:

- **transfer** spin information over centimeter distances
- **process** the information (using wave nature of magnon)
- **operate** in insulator-based technology

Fundamental properties:

- Minimal wavelength is down to several nm
- Frequency is in GHz and up to the THz range
- Energy: $E_{magnon} << k_B T$
- Lifetime: up to several 100 ns

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Solving the Landau-Lifshitz equation

1st step:

solve LL equation of motion

$$\frac{1}{\gamma}\frac{\mathrm{d}\vec{M}}{\mathrm{d}t} = -\vec{M} \times \vec{B}_{eff}$$

⇒ 6 partial wave solutions

- non-linearity intrinsically built in due to $M_i \cdot B_{\mathrm{eff},j}$ products

2nd step:

formulate boundary conditions

⇒ coupled partial waves

a) boundary conditions from Maxwell equations

b) Rado-Weertman boundary condition
(from LL-equation)

$$\vec{M} \times \left(A \frac{\partial}{\partial \vec{n}} \vec{M} - k_s \vec{n} \left(\vec{n} \cdot \vec{M} \right) \right) = 0$$

with:

$\vec{n}$:	unit vector normal to surface
$\partial \vec{M} / \partial \vec{n}$:	derivative of $\vec{M}$ in direction of $\vec{n}$
k_s:	out-of-plane interface anisotropy constant

Magnetostatic dipolar modes: Approximations

Approximation: ultrathin films ($q_{||} \cdot d \ll 1$):

Damon-Eshbach mode with frequency

$$\left(\frac{\omega}{\gamma}\right)^2 = \left(B_0 + J_s\left(1 - \tfrac{1}{2} q_{||} d\right) + \frac{\partial^2 E_{\text{ani}}}{M_s\, \partial \theta^2}\right)\left(B_0 + \frac{\partial^2 E_{\text{ani}}}{M_s\, \partial \phi^2} + \tfrac{1}{2} J_s q_{||} d \sin(\phi - \phi_0)\right)$$

with

E_{ani} : free anisotropy energy

θ, ϕ : polar and azimuthal angle

$\phi - \phi_0$: angle between external field and magnetization

Free anisotropy energy and thus magnetic anisotropies determine spin wave frequencies
⇒ determination of anisotropy properties in magnetic saturation

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Lecture 1 - CONTENT

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

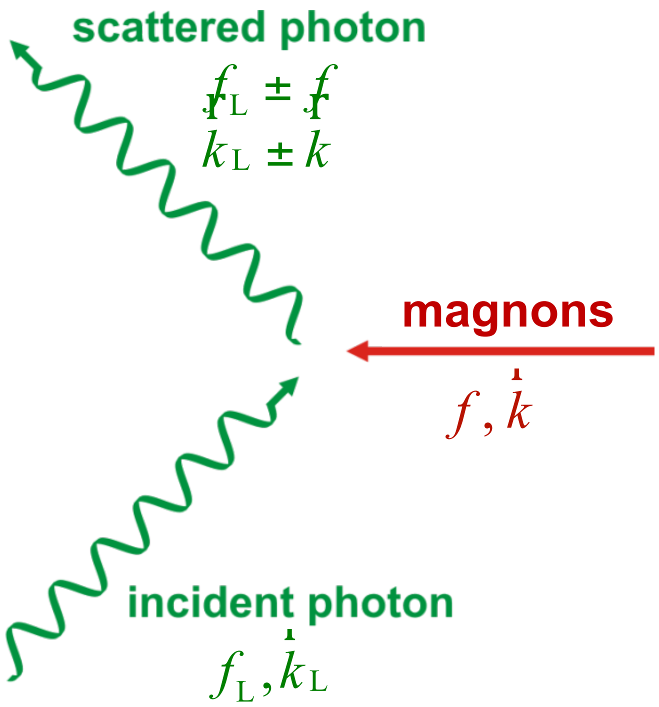
Technische Universität Kaiserslautern

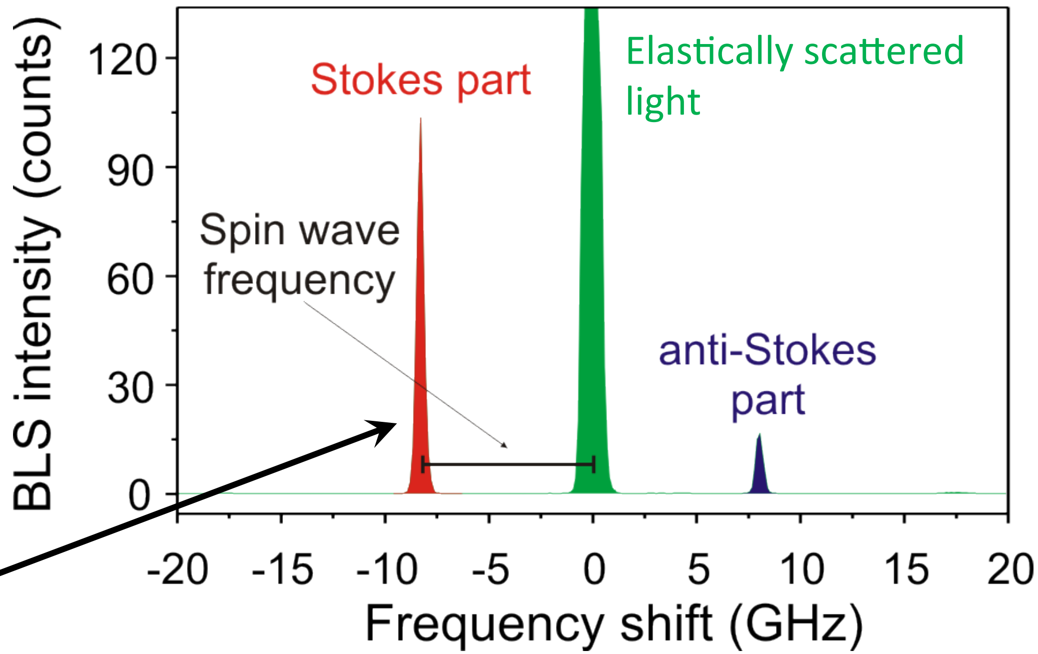
Brillouin light scattering spectroscopy

Brillouin light scattering process

= inelastic scattering of photons from spin waves

$$f_{\text{scattered L}} = f_L \pm f$$
$$\vec{k}_{\text{scattered L}} = \vec{k}_L \pm \vec{k}$$

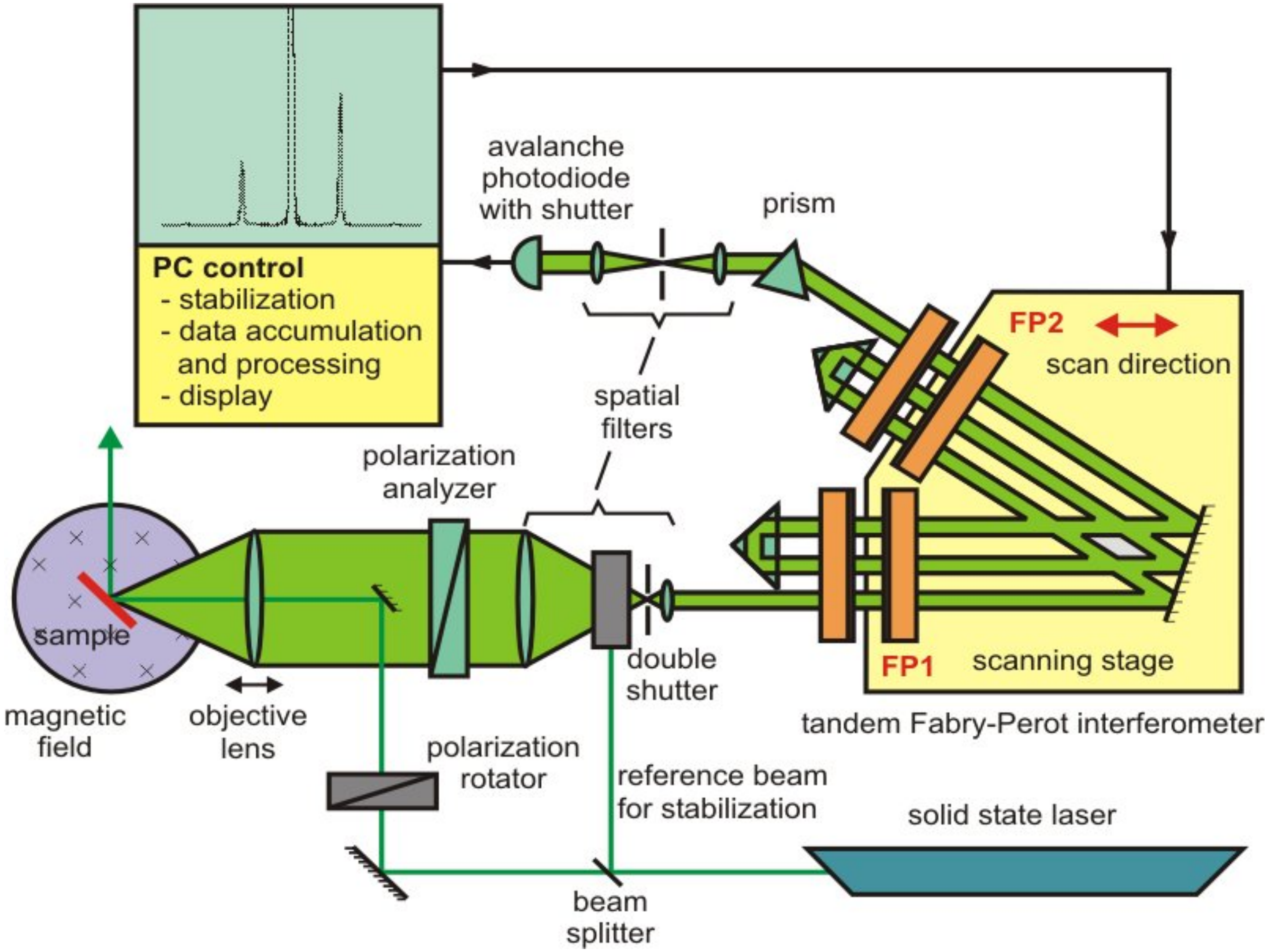




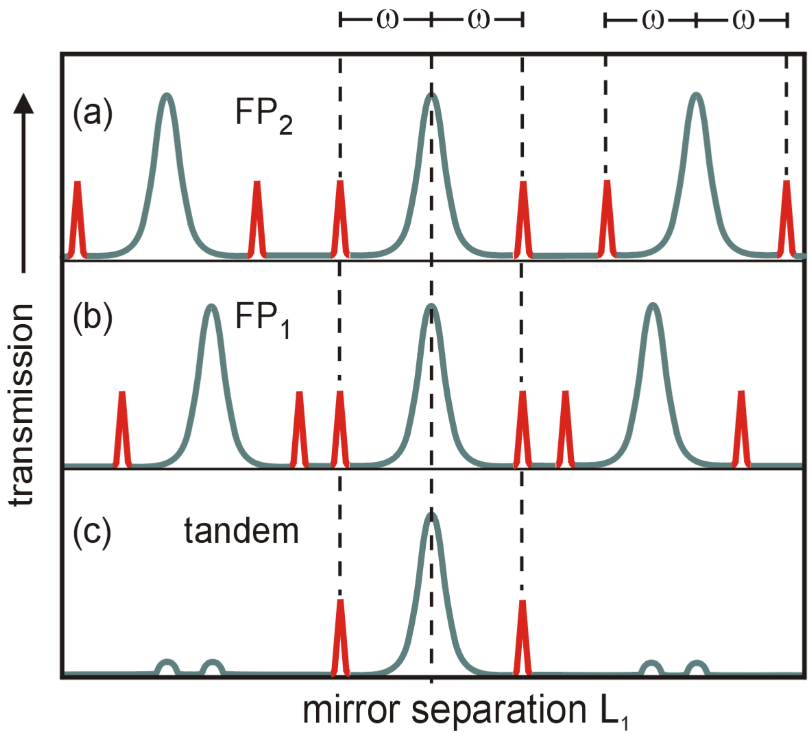
proportional to the spin wave intensity $|\varphi|^2$

Brillouin light scattering spectrometer

high-resolution interferometry with high contrast for measurements of acoustic phonons and spin waves



Brillouin light scattering spectrometer



- etalon in transmission if mirror separation *L* is:

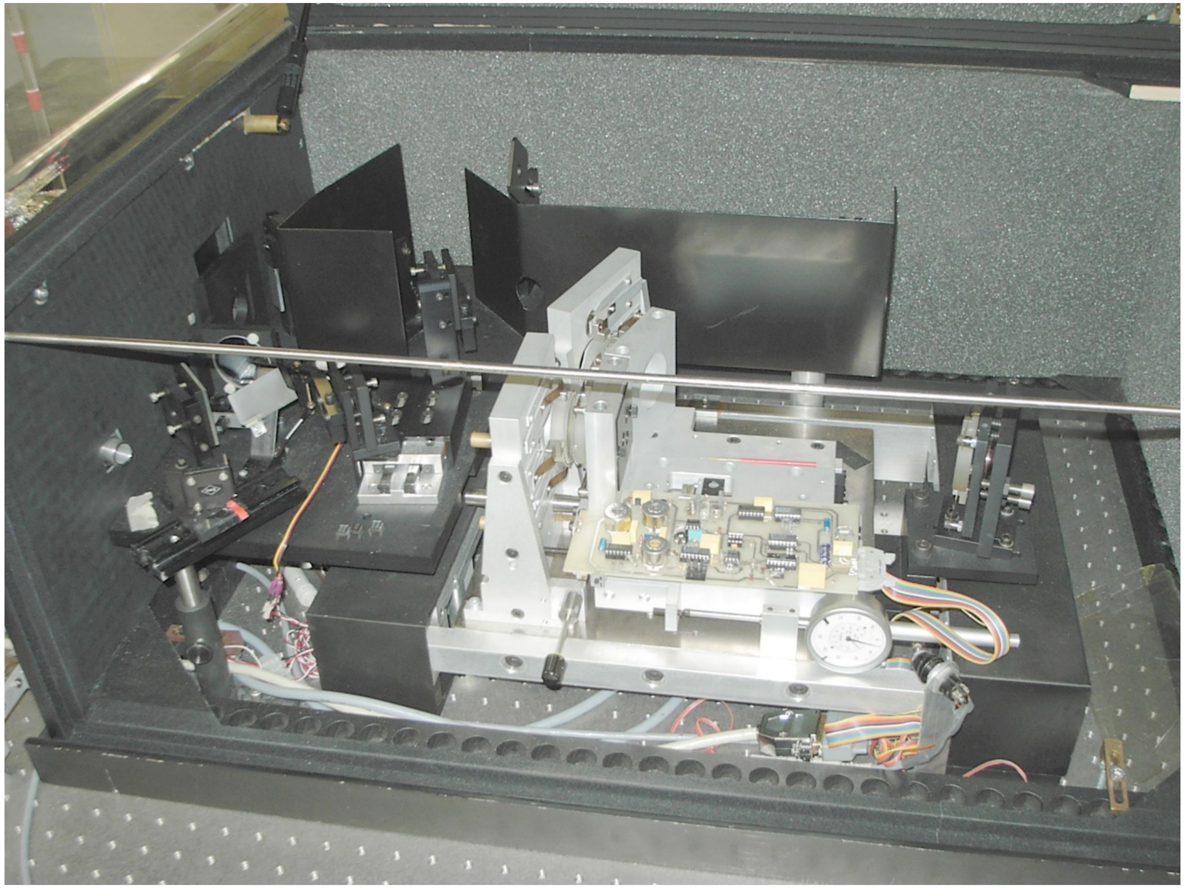
  $L = n\, \lambda_{Laser}/2$

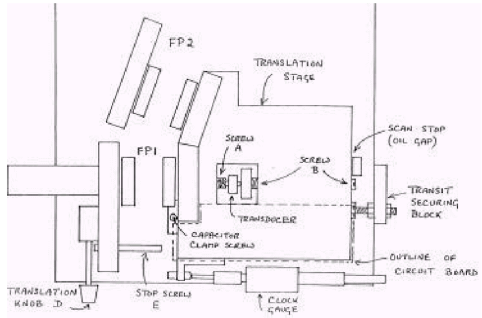
- suppression of neighboring orders if mirror separations L_1, L_2 of both etalons:

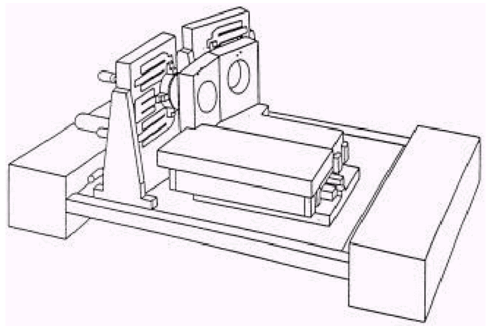
  $L_2 = L_1 \cos \alpha$

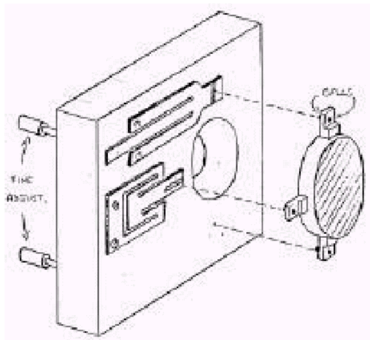
  α : angle between etalon axes

Brillouin light scattering spectrometer





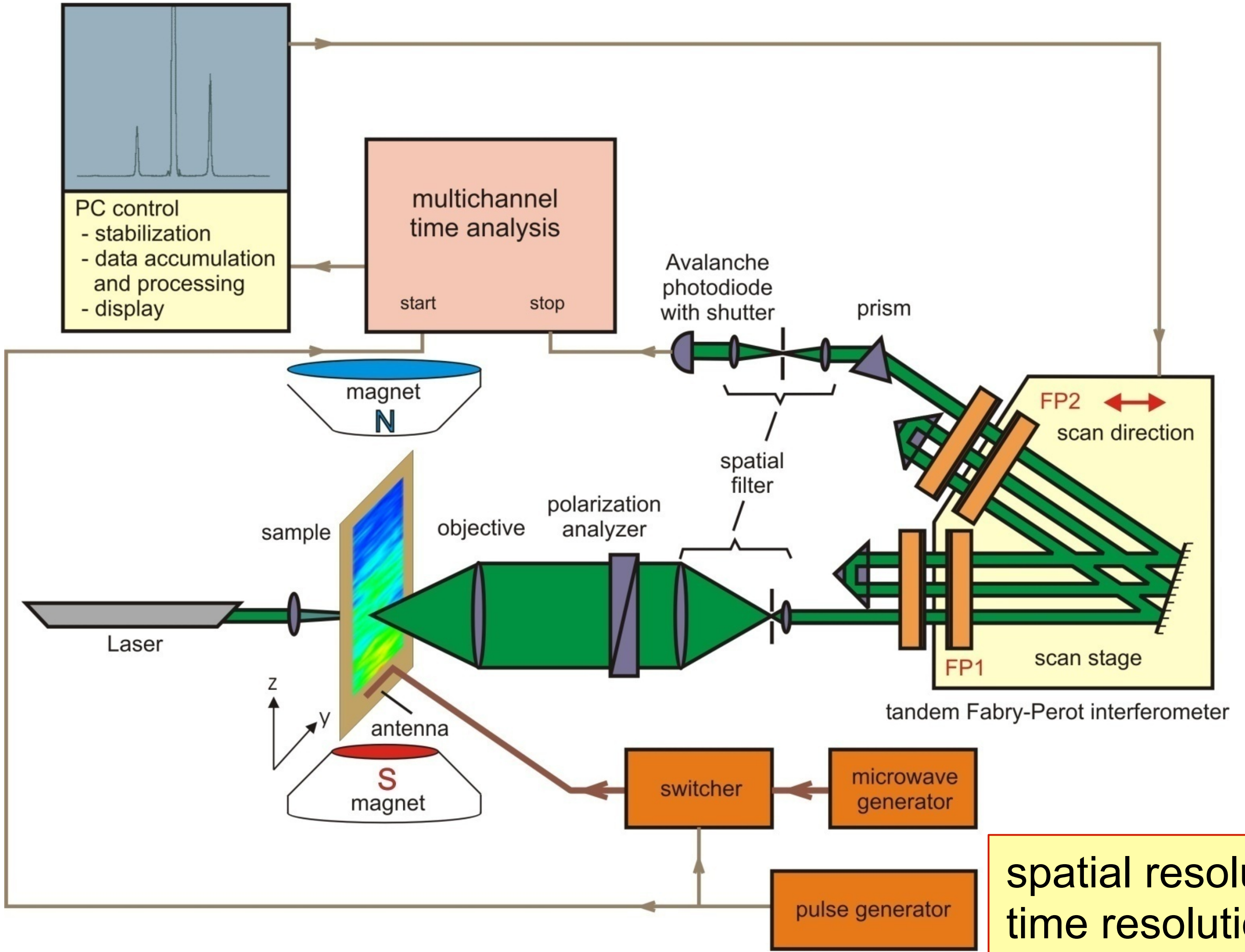




Tandem Fabry-Perot Interferometer

Sketch of mechanical stage and mirror mounts
(from John Sandercock's 1993 manual)

Space and time resolved BLS



O. Büttner *et al.*, PRB **61**, 11576 (2000)

spatial resolution: 40 µm
time resolution: 1 ns
dynamic range: >60 dB

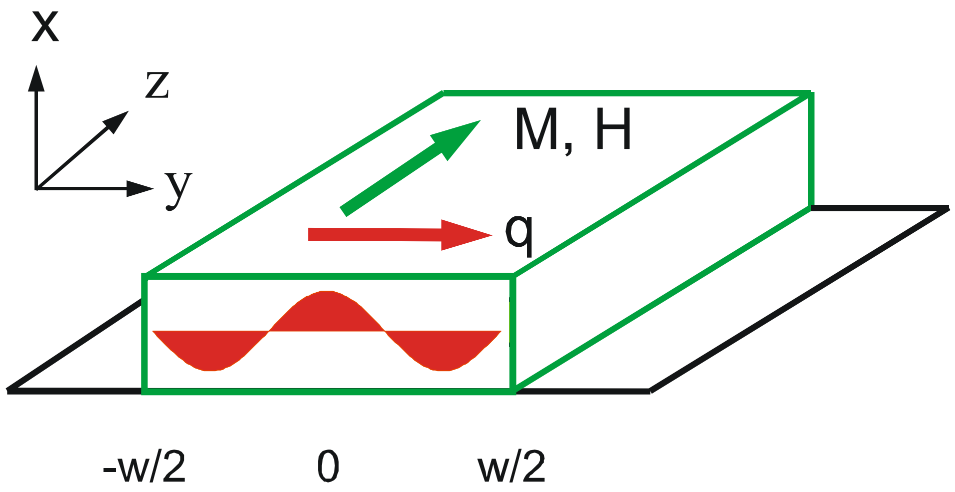
Lecture 1 - CONTENT

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

Magnetization dynamics

Confinement to magnetic objects:

quantized eigen modes („standing spin waves“)



➔ Find dynamic ground state, i.e., eigenmode spectrum

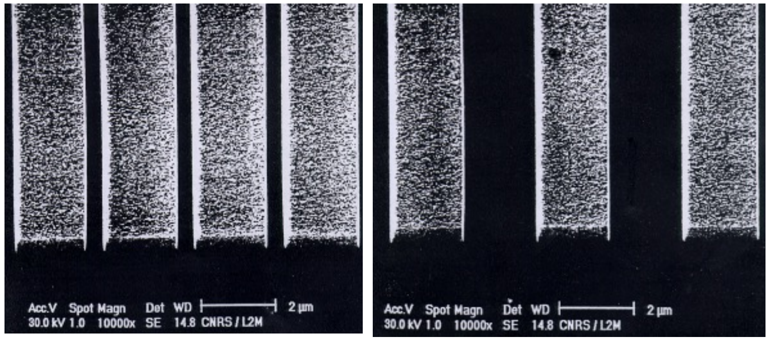
Problems:

- correct boundary conditions
- modes in inhomogeneously magnetized structures

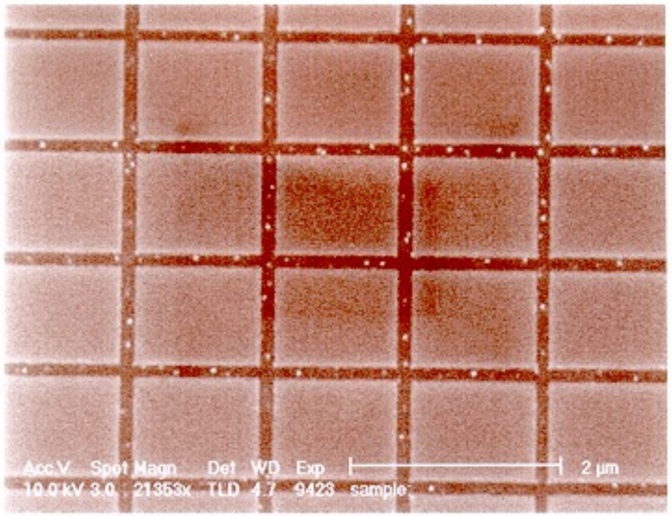
Patterned magnetic films

Au / $Ni_{81}Fe_{19}$ (220nm) / SiO_2 / Si
preparation: e-beam evaporation in UHV
coercivity: H_c = 1-2 Oe
patterning: x-ray lithography (LURE, France)

Wires:

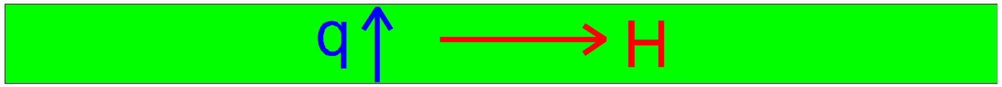


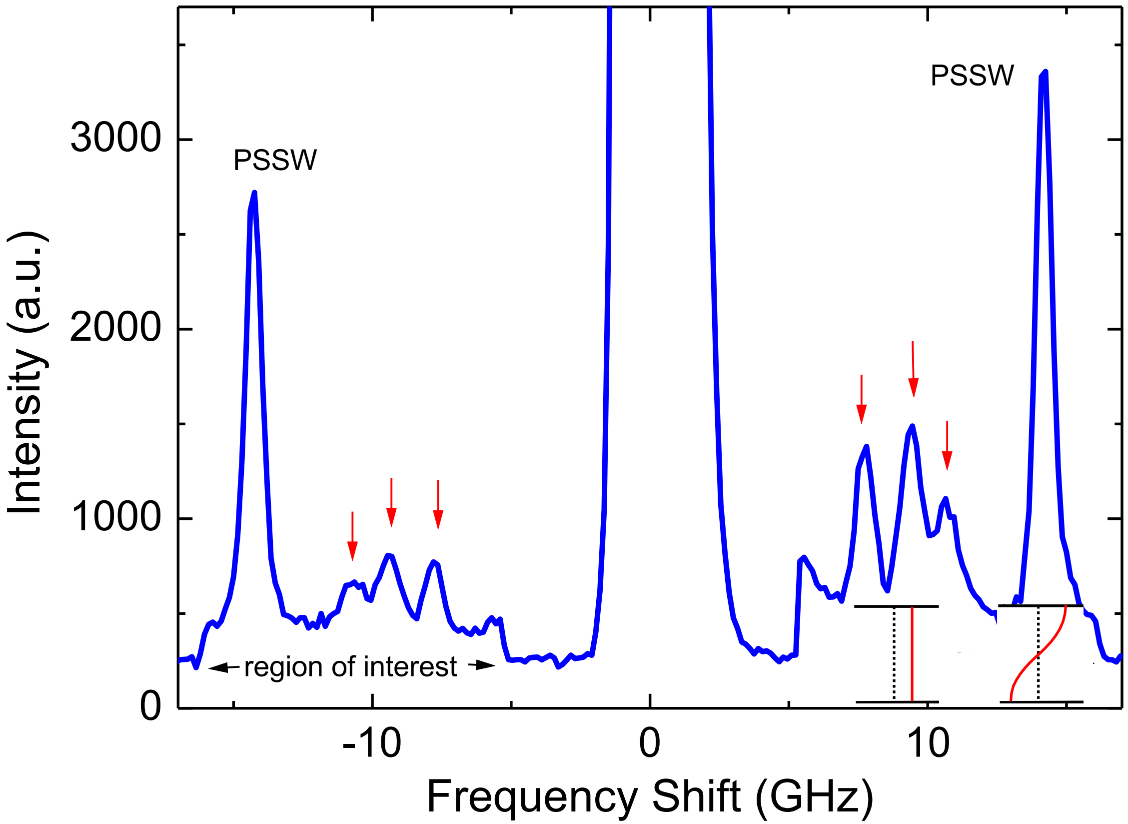
Dots:



Light scattering in magnetic wires

- Damon-Eshbach mode geometry:

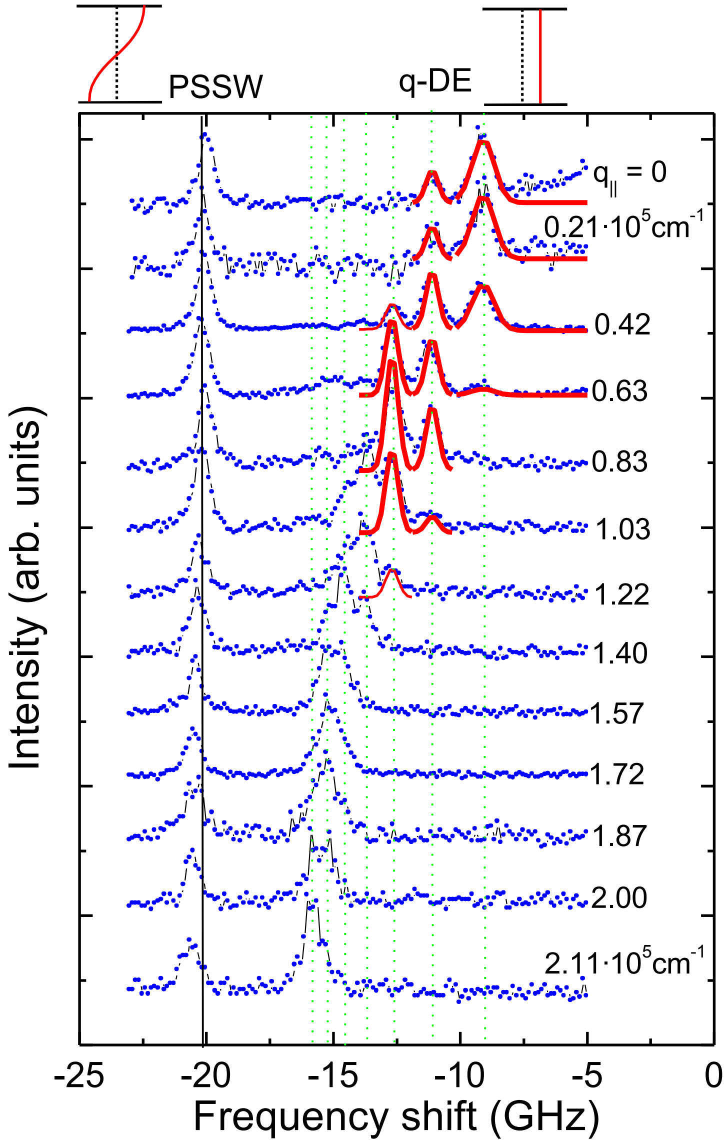




many modes with the same transferred wavevector but with different frequencies

Spin wave spectra

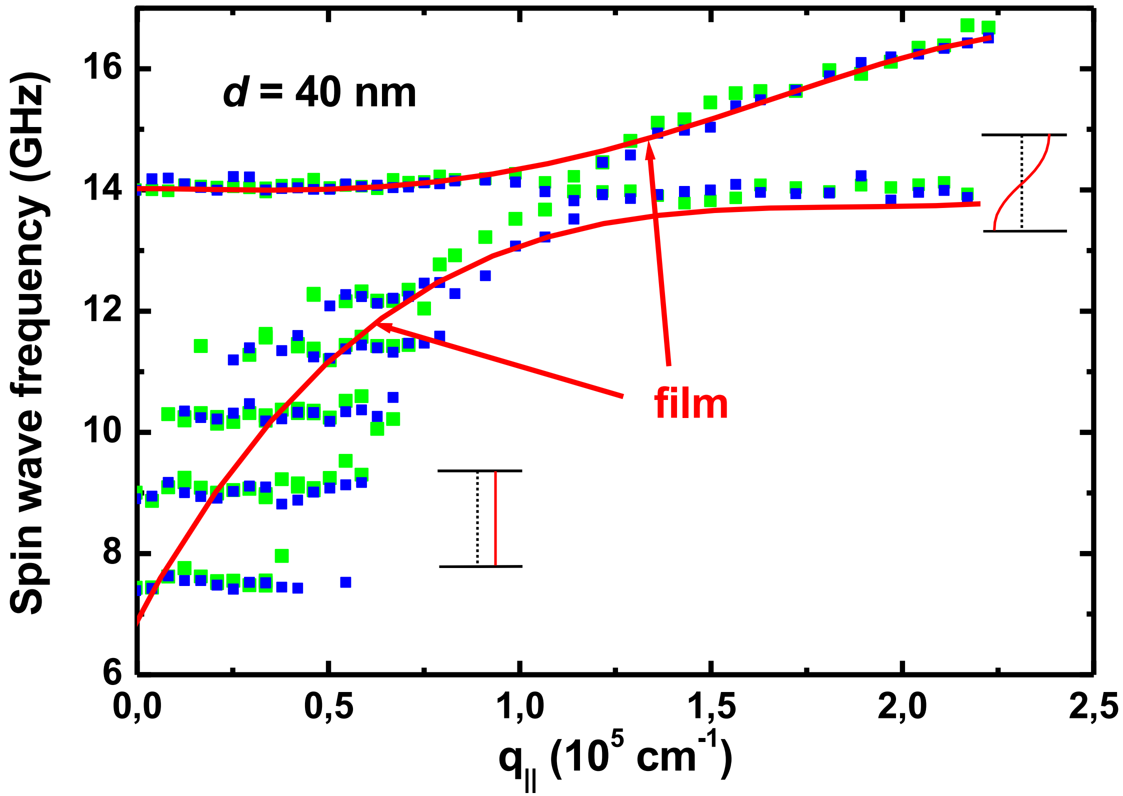
35 mm NiFe wires
1.75/0.3 µm

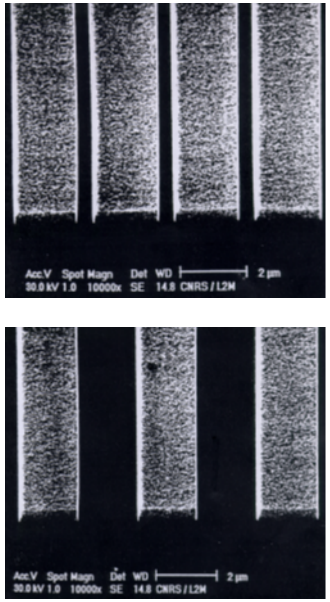


localized
dispersionless modes

Technische Universität Kaiserslautern

Dispersion





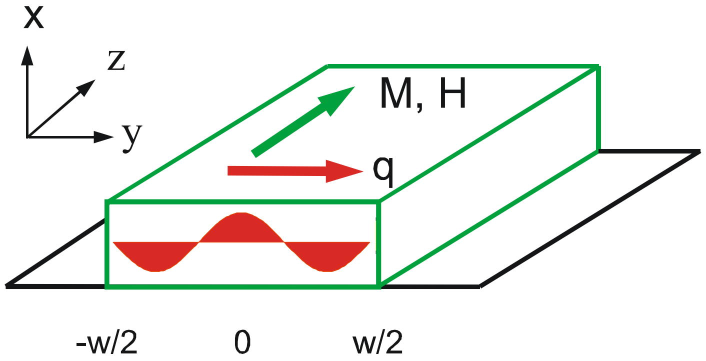
40 nm NiFe stripes

- ■ 1.8/0.7 µm
- ■ 1.8/2.2 µm

no interaction, single-stripe effect
spin wave quantization in a single stripe

Lateral quantum size effect

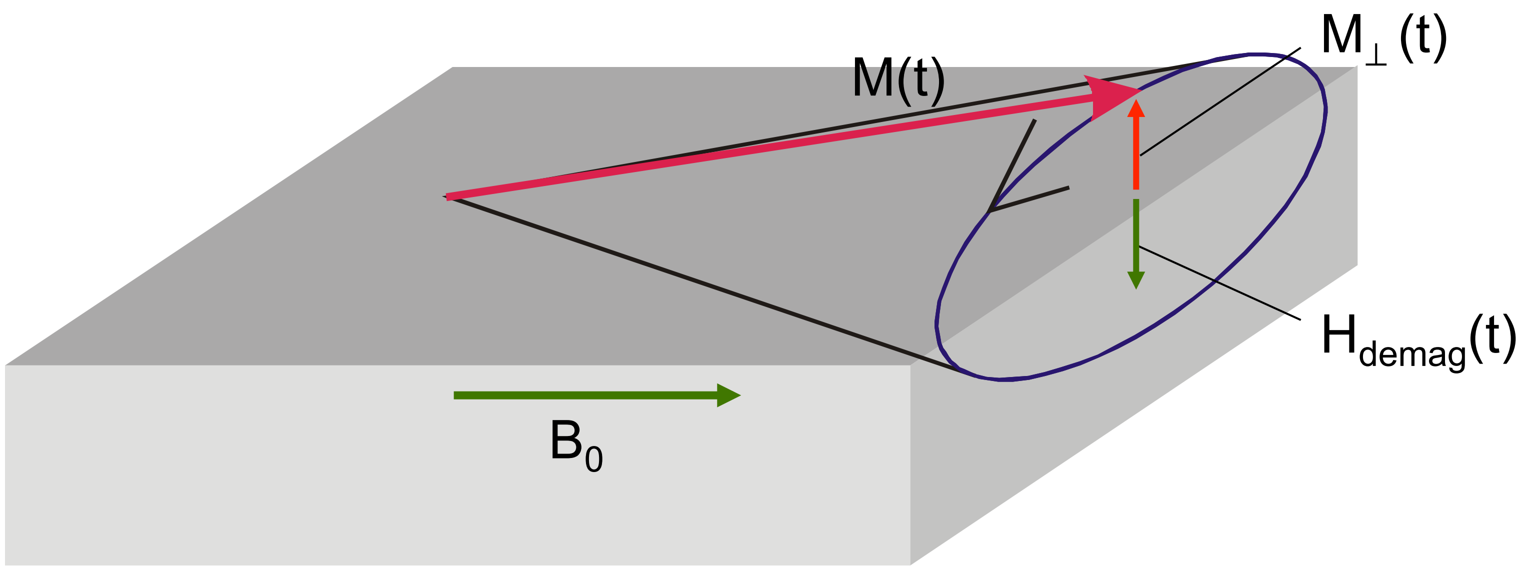
Standing lateral modes:



Standing lateral modes

- propagating dipolar modes (Damon-Eshbach modes) perpendicular to wires: "standing lateral modes"
- quantization condition due to the lateral edges:
  $w = n\,\lambda_{\text{spin wave}}/2$;
  $q_n = 2\pi/\lambda_{\text{spin wave}} = n\pi/w$; $n = 1,2,3,...$
- boundary conditions (open – pinned)
  - take dynamic stray fields into account
- calculation of frequencies by inserting q_n into Damon-Eshbach equation of motion

Boundary conditions for dynamic magnetization



Precessing magnetization has dynamic out-of-plane component
⇒ dynamic stray fields and thus dynamic surface torque on magnetization

Mode profiles

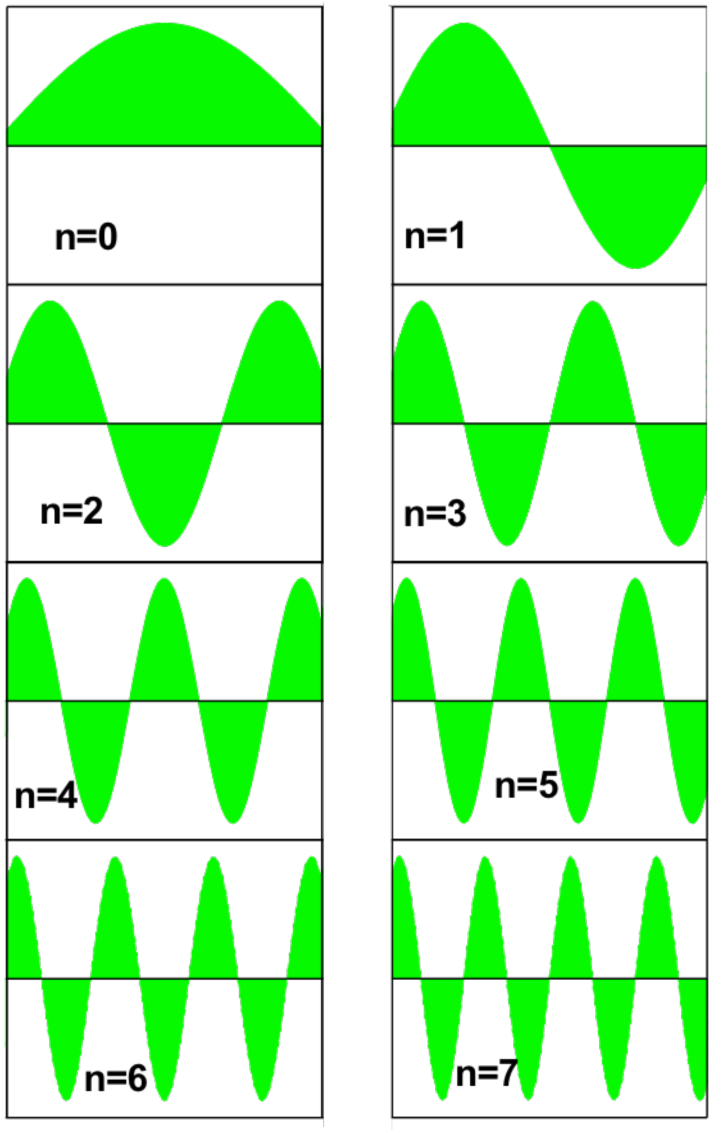
New dynamic dipole boundary condition for non-elliptical elements:

$$\frac{\partial \vec{m}}{\partial \hat{n}} + \frac{1}{\xi_D}\vec{m} = 0$$

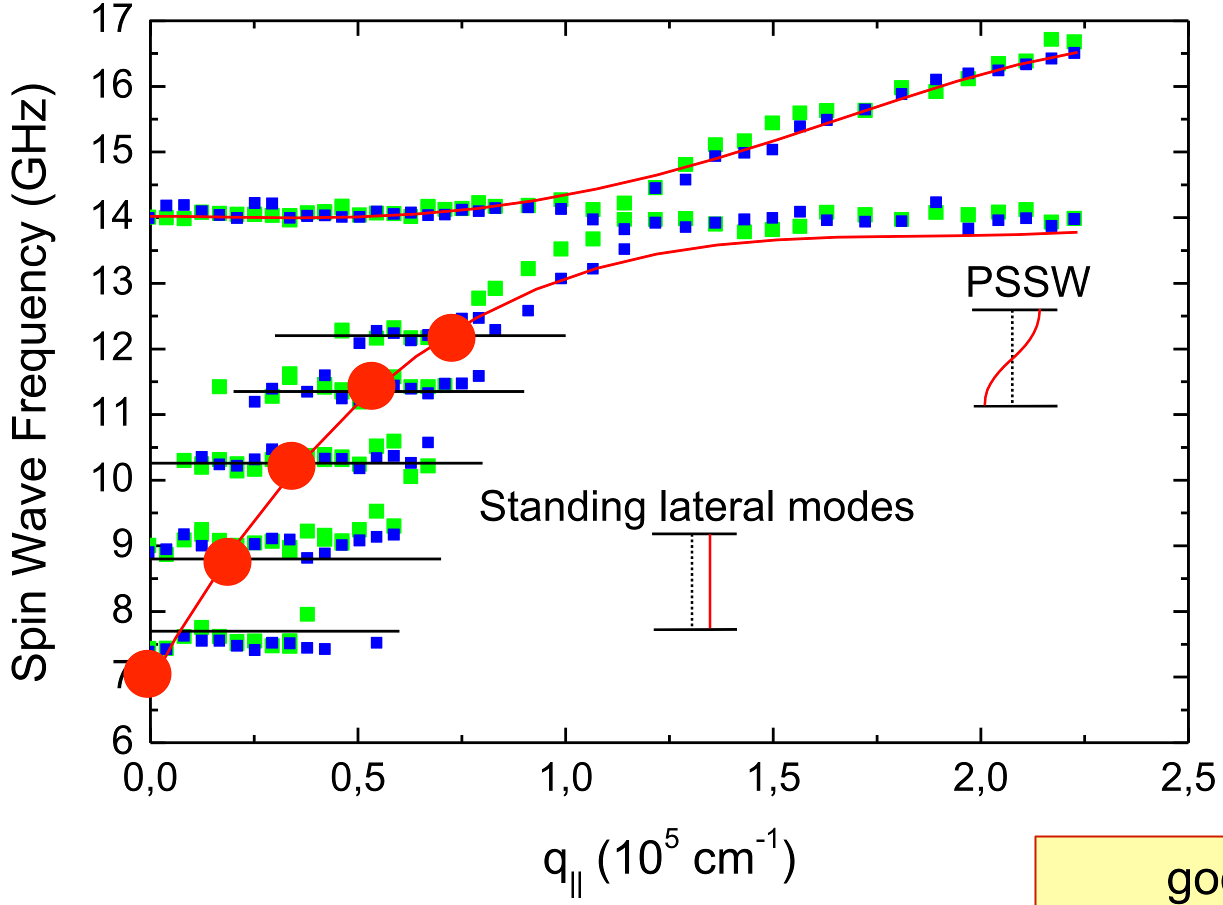
$$\xi_D = \frac{t}{2\pi}\left(1 + 2\ln\frac{w}{t}\right)$$

- Takes dynamic stray fields into account
- „Stray field induced pinning“

low-index modes ($\lambda \gg \xi_D$)	„pinned“
high-index modes ($\lambda \approx \xi_D$)	„unpinned“



Frequencies of the quantized modes



width:

1.8 µm

separation:

0.7 µm (green)

2.2 µm (blue)

thickness: 50 nm

good quantitative agreement between the theory and the experiment is obtained

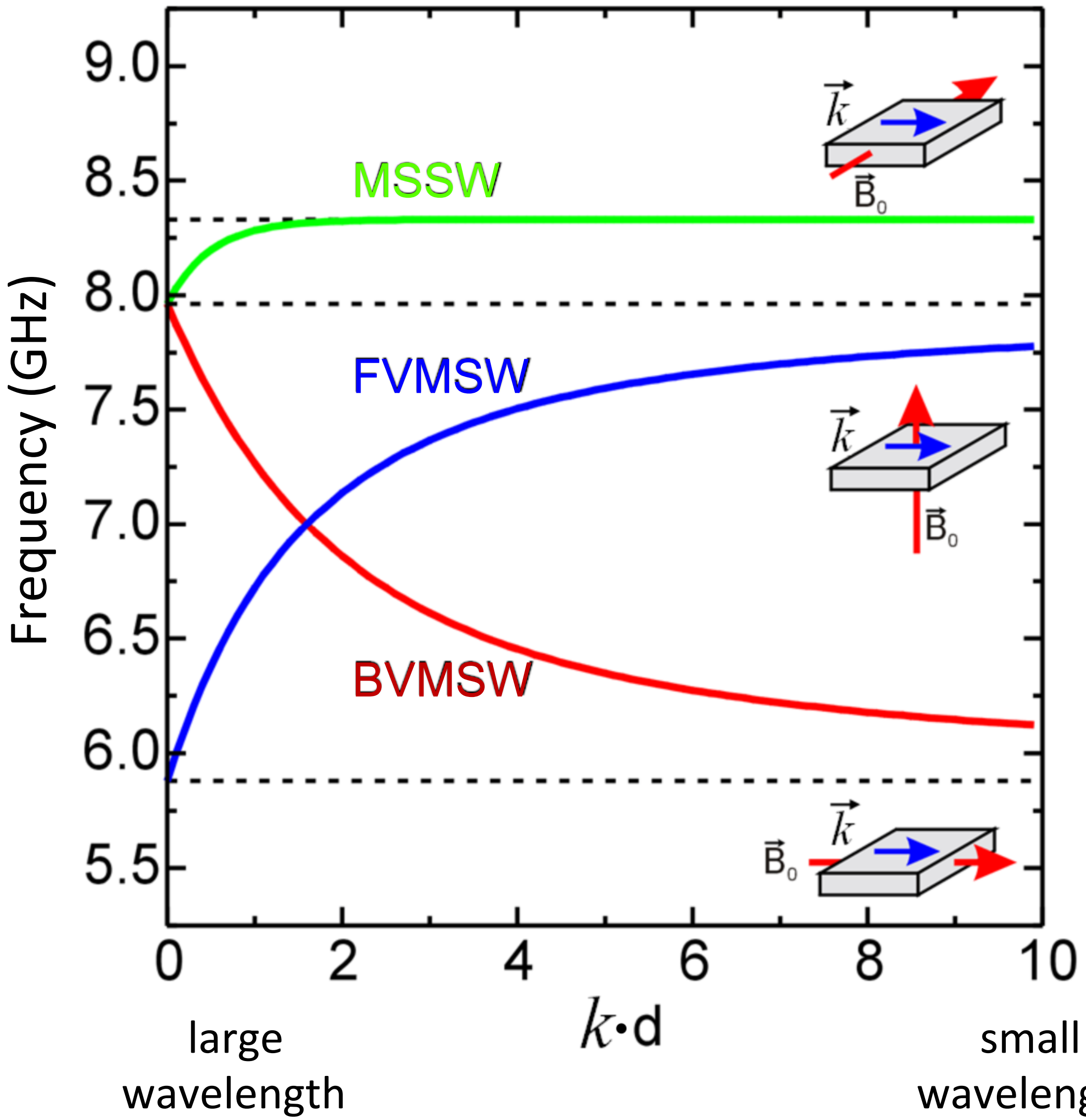
C. Mathieu et al., PRL **81,** 3968 (1998)

Lecture 1 - CONTENT

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Dipolar spin waves

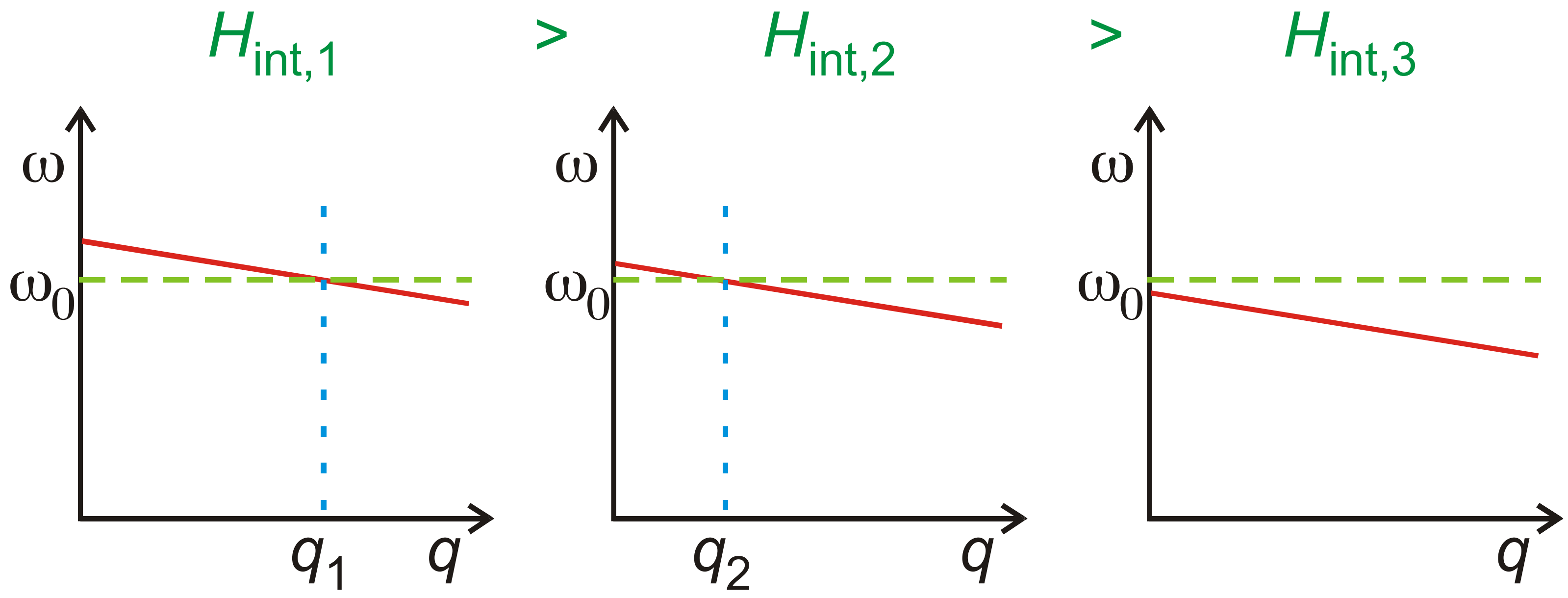


Magnetostatic Surface Spin Waves

Forward Volume Magnetostatic Spin Waves

Backward Volume Magnetostatic Spin Waves

Motion of a spin wave packet in varying field

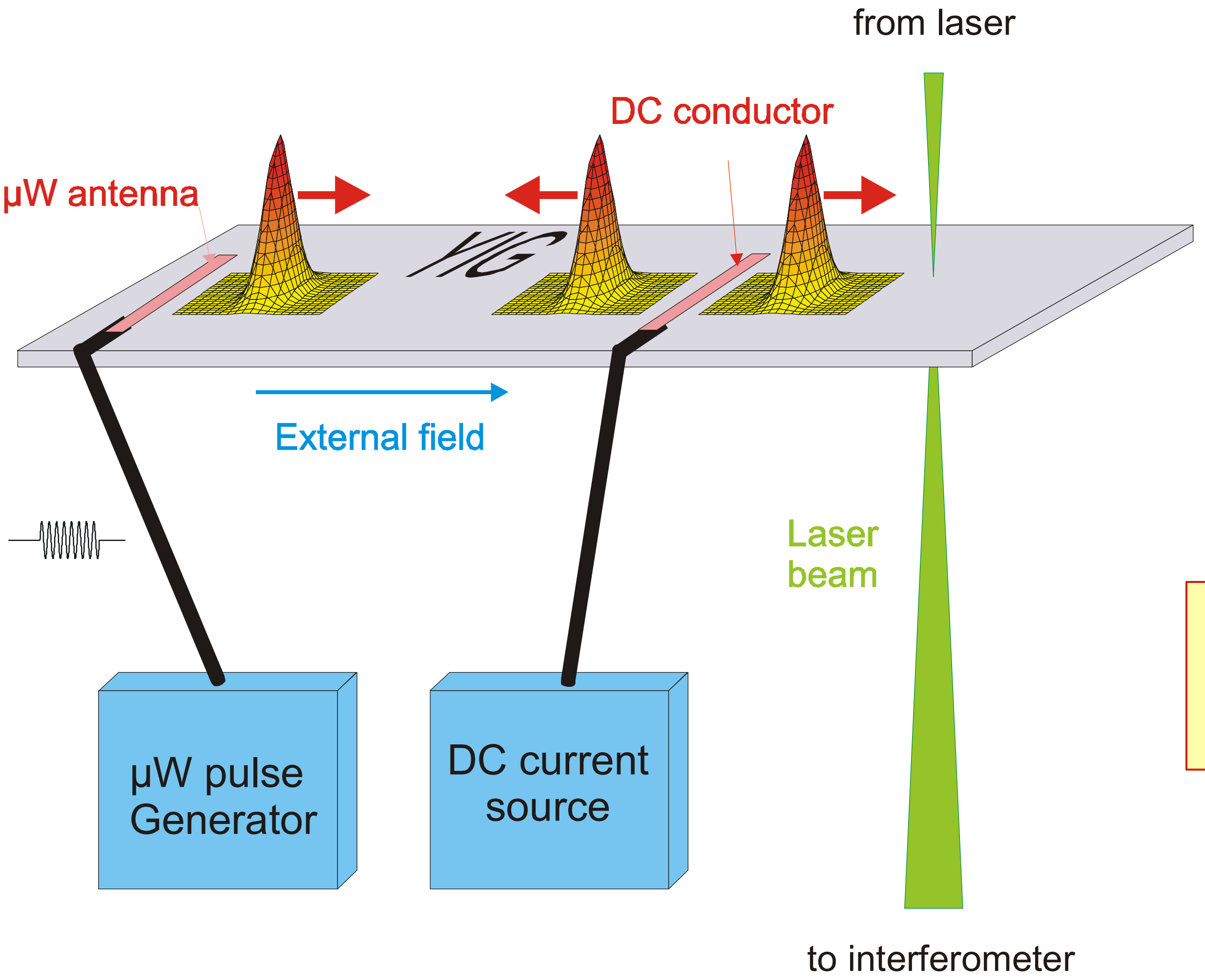


propagation

propagation ($q_2 < q_1$)

reflection

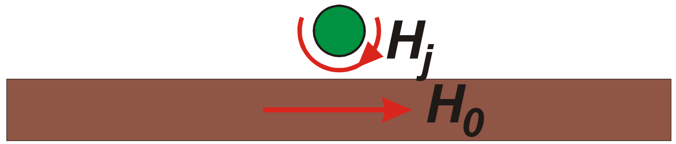
Real-time observation of spin wave propagation



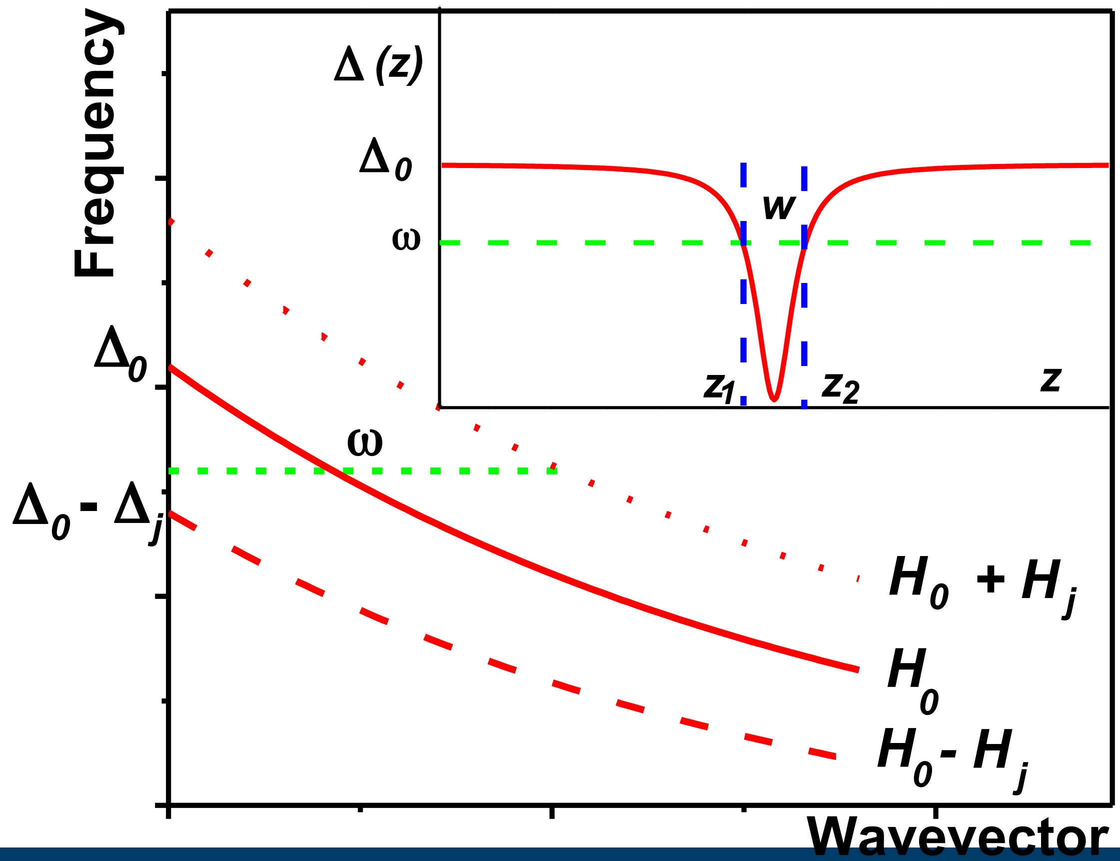
SW-pulses created by microwaves and detected by light scattering with time and space resolution

DC conductor provides a local field inhomogeneity

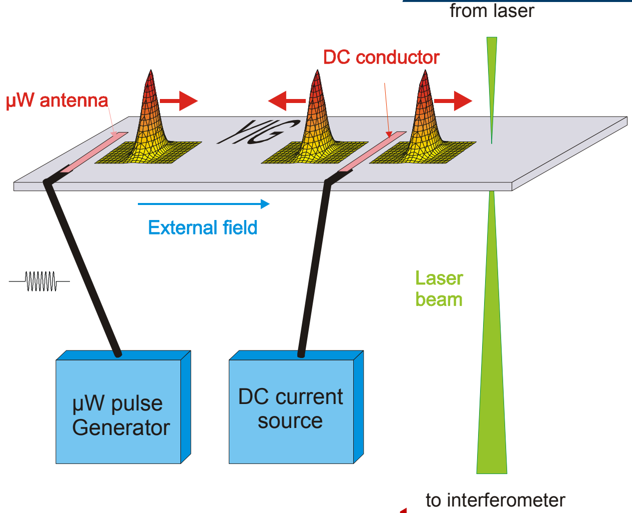
Spin wave tunneling



Δ: zero-wavevector gap

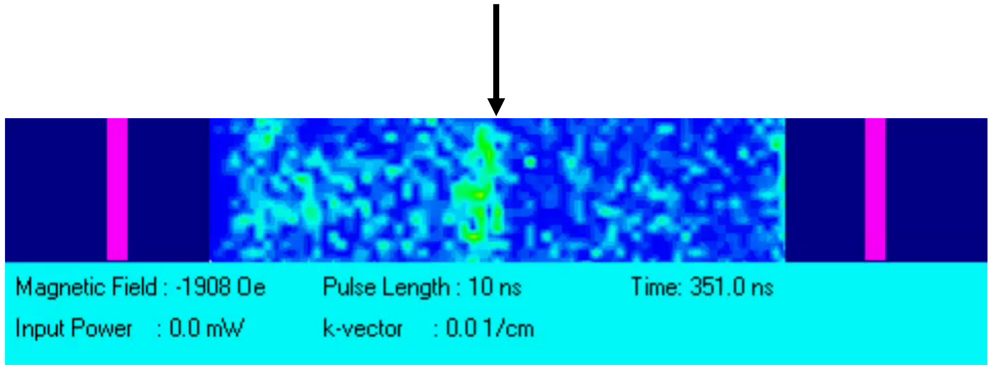


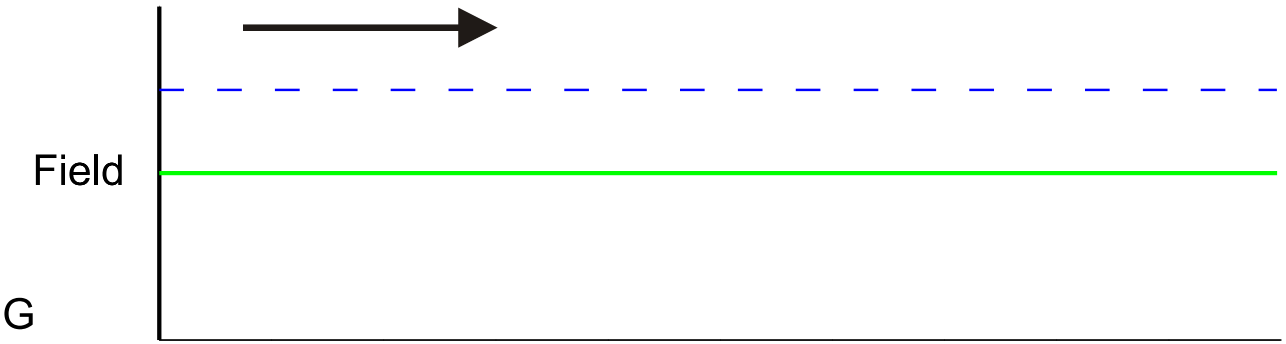
Spin wave pulse propagation



zero current

position of the dc conductor

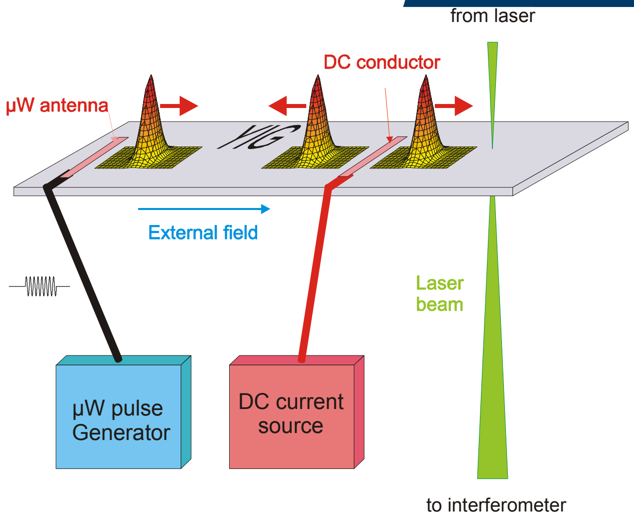




magnetic material: YIG

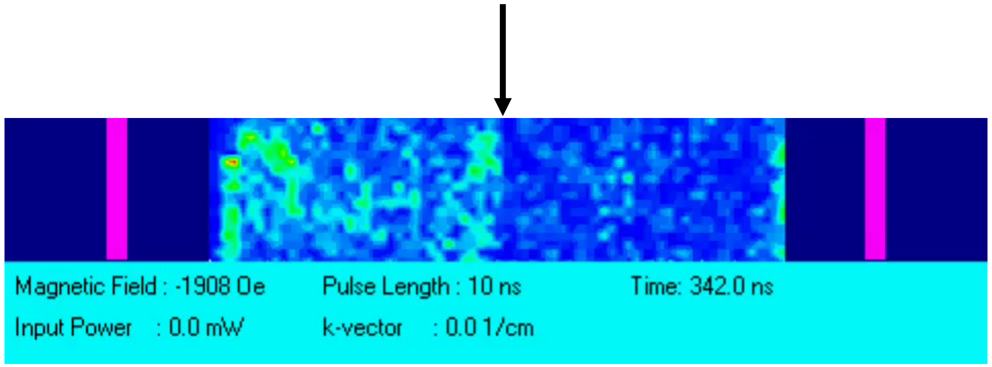
S.O. Demokritov et al., Phys. Rev. Lett. **93**, 047201 (2004)

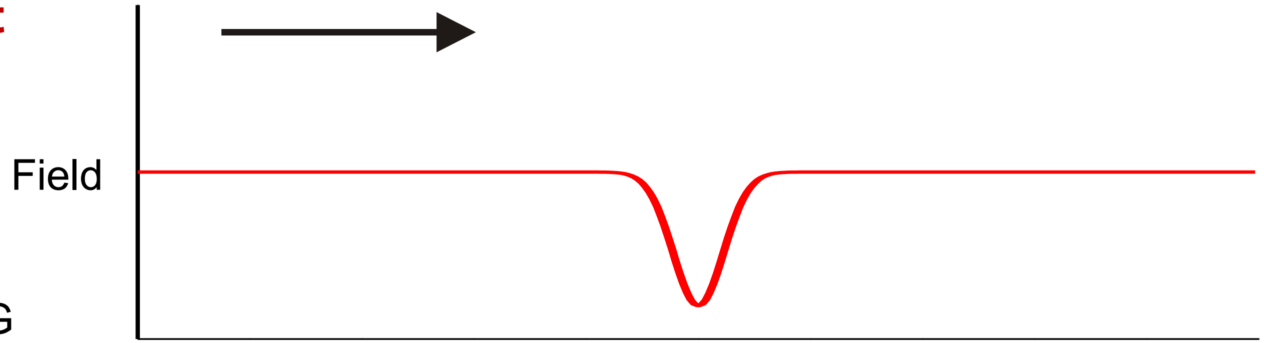
Spin wave pulse propagation



positive current

position of the dc conductor





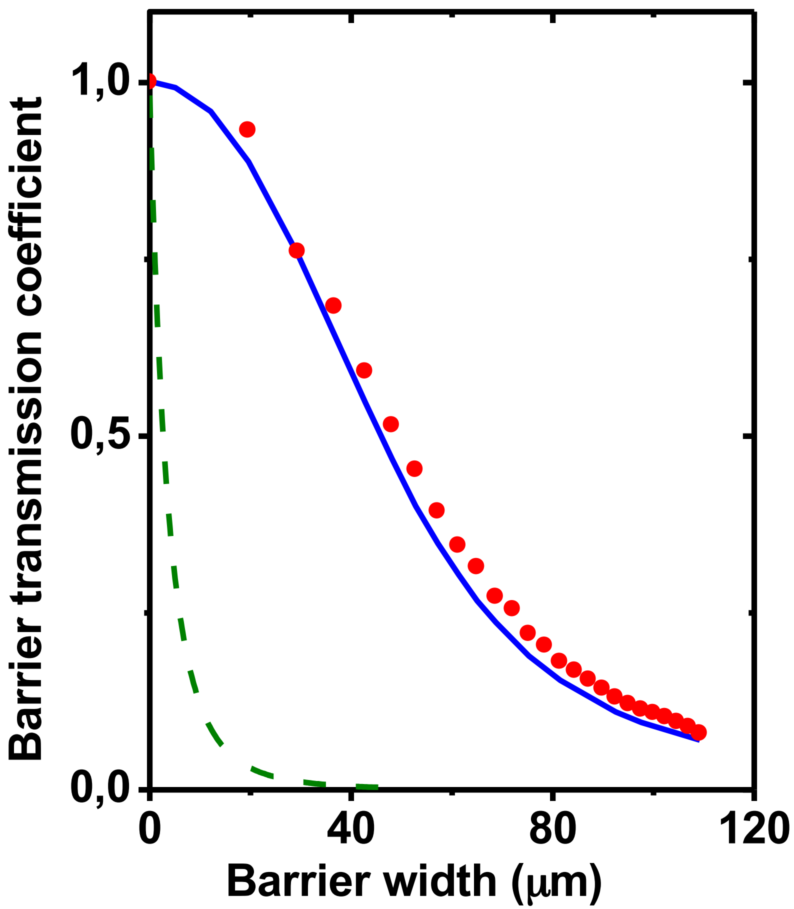
magnetic material: YIG

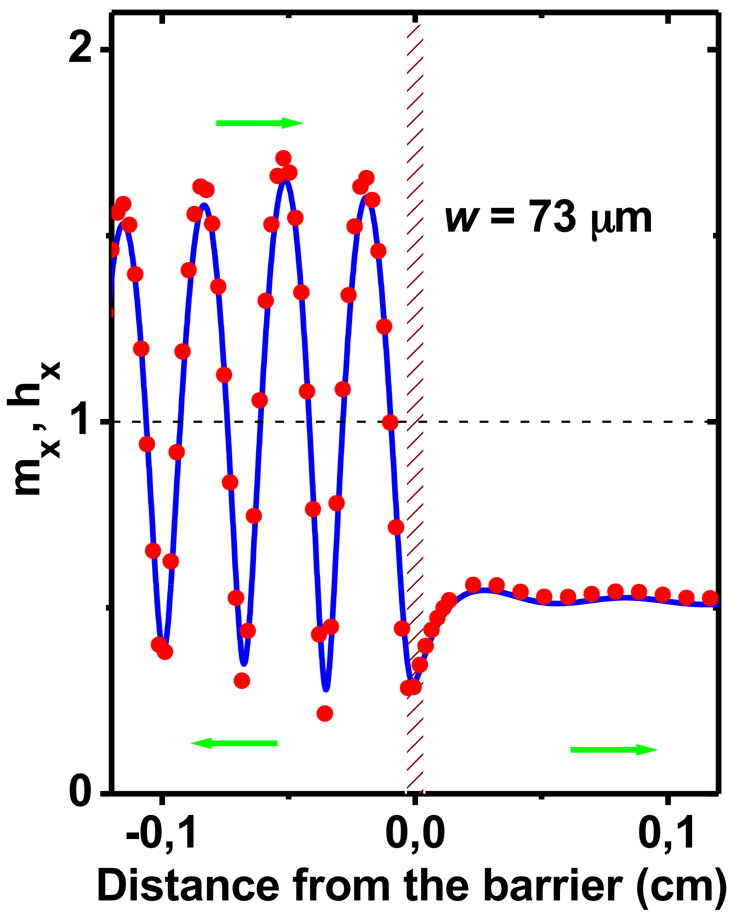
dip in field acts like potential barrier

Potential barrier: reflection and tunneling

S.O. Demokritov et al., Phys. Rev. Lett. **93**, 047201 (2004)

Spin wave tunneling





Non-exponential decrease of spin wave intensity with barrier size

Reflection of spin wave at barrier and spin wave tunneling

Carrier frequency:
7.125 GHz

Bias field:
1836 Oe

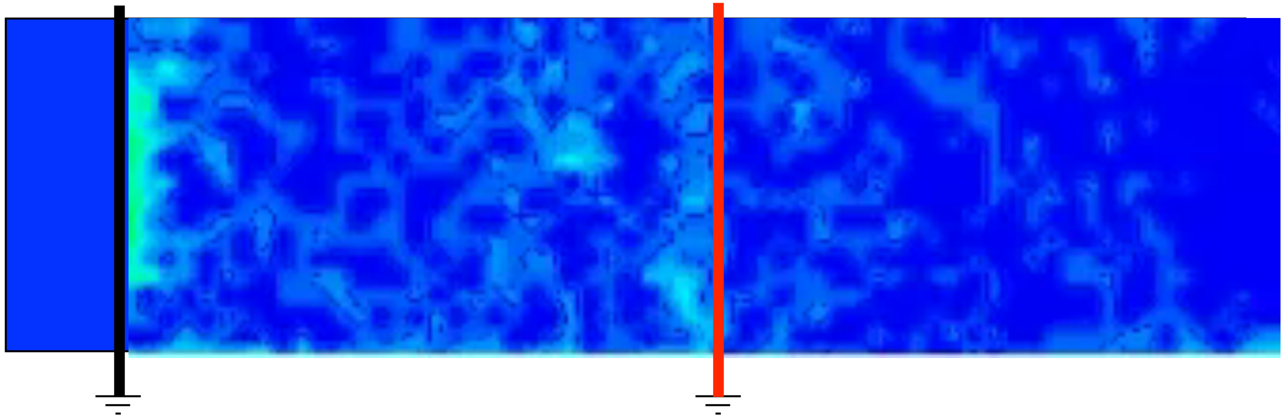
Wave number:
112 rad/cm

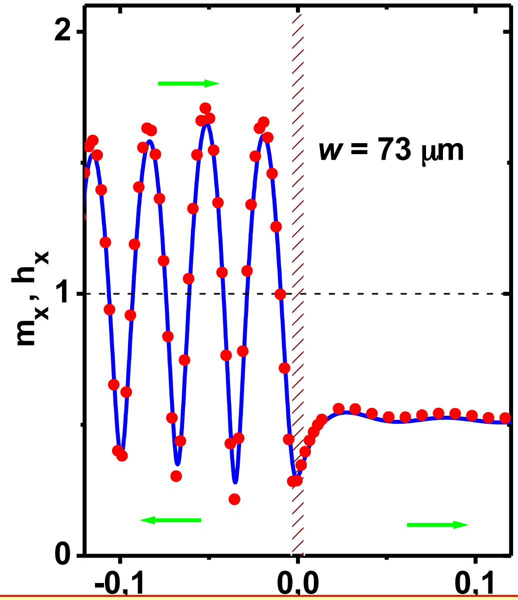
Group velocity:
≈30 km/s

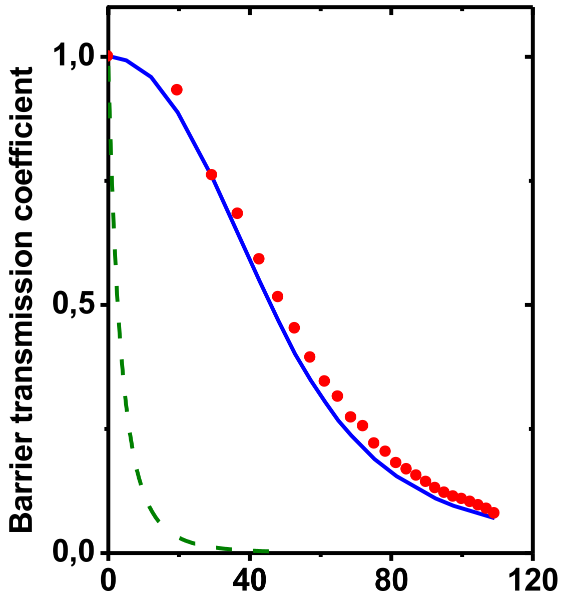
Film thickness:
5.7 μm

Scan region:
6.0 × 1.8 mm²

Logarithmic scale







Non-exponential decrease of spin wave intensity with barrier size

Spin wave Fabry-Perot

Carrier frequency:
7.125 GHz

Bias field:
1836 Oe

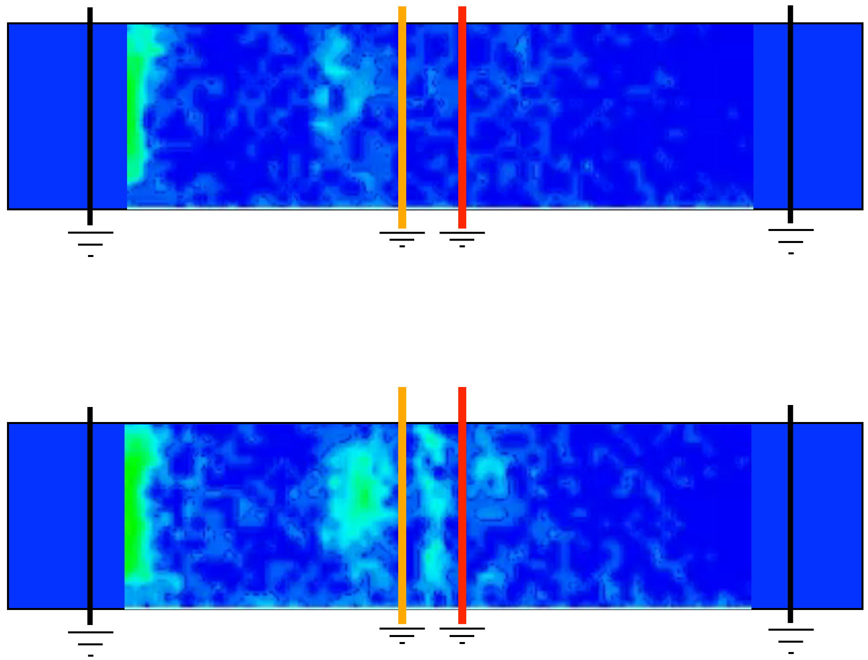
Wave number:
112 rad/cm

Group velocity:
≈30 km/s

Film thickness:
5.7 µm

Scan region:
6.0 ×1.8 mm^2

Logarithmic scale



Short SW pulse
18 ns

Long SW pulse
40 ns

Spin wave tunneling through mechanical gap

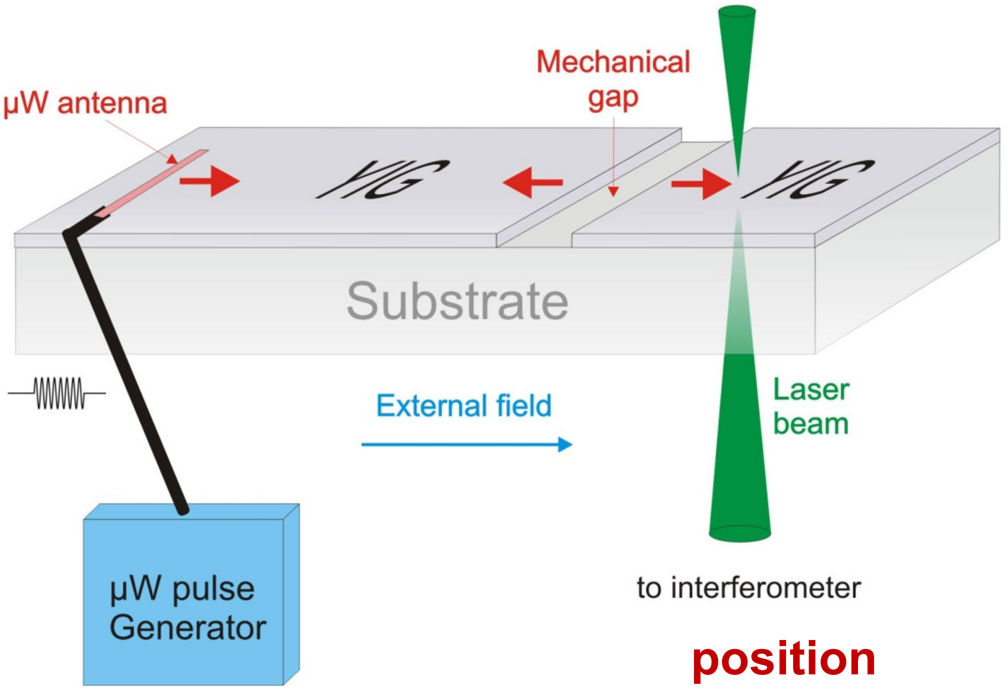
Film thickness:
6 µm

Gap width:
20 µm

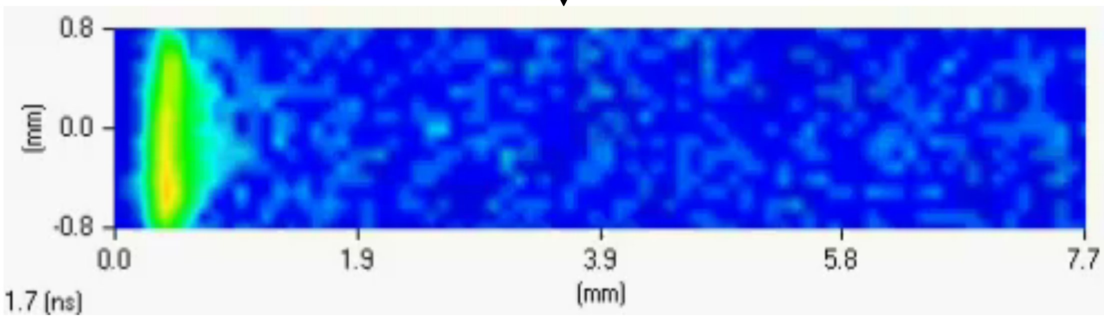
Frequency:
7.125 GHz

Magnetic field:
1835 Oe

Logarithmic scale



position of the gap



Spin wave cavity

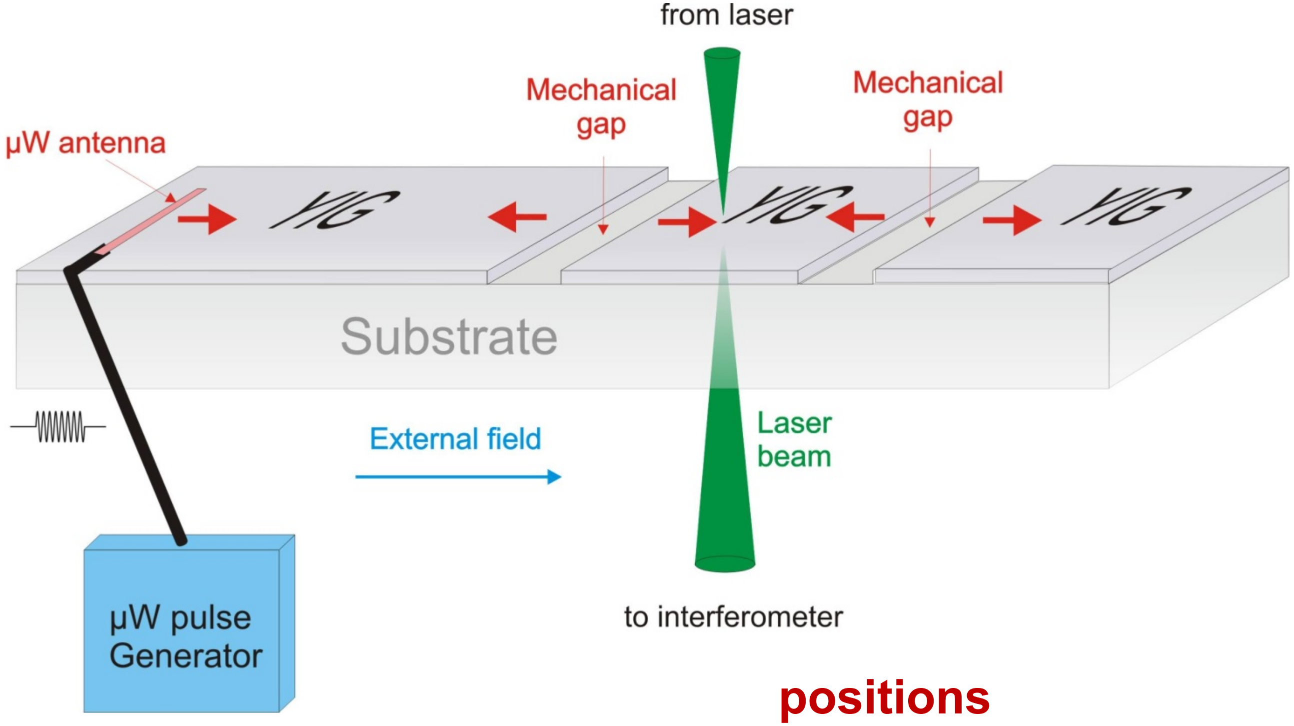
Film thickness:
6 µm

Gap width:
20 µm

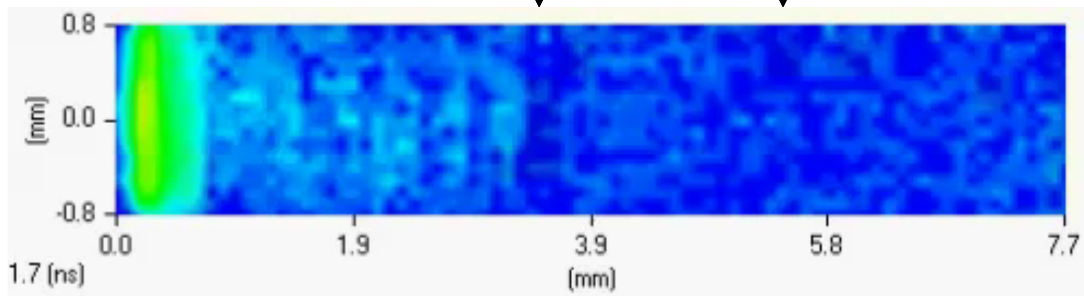
Frequency:
7.125 GHz

Magnetic field:
1839 Oe

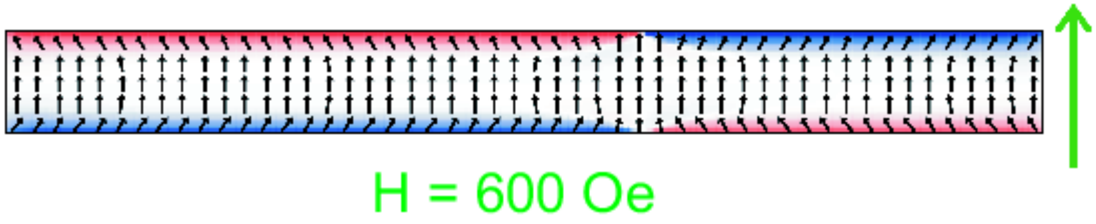
Logarithmic scale

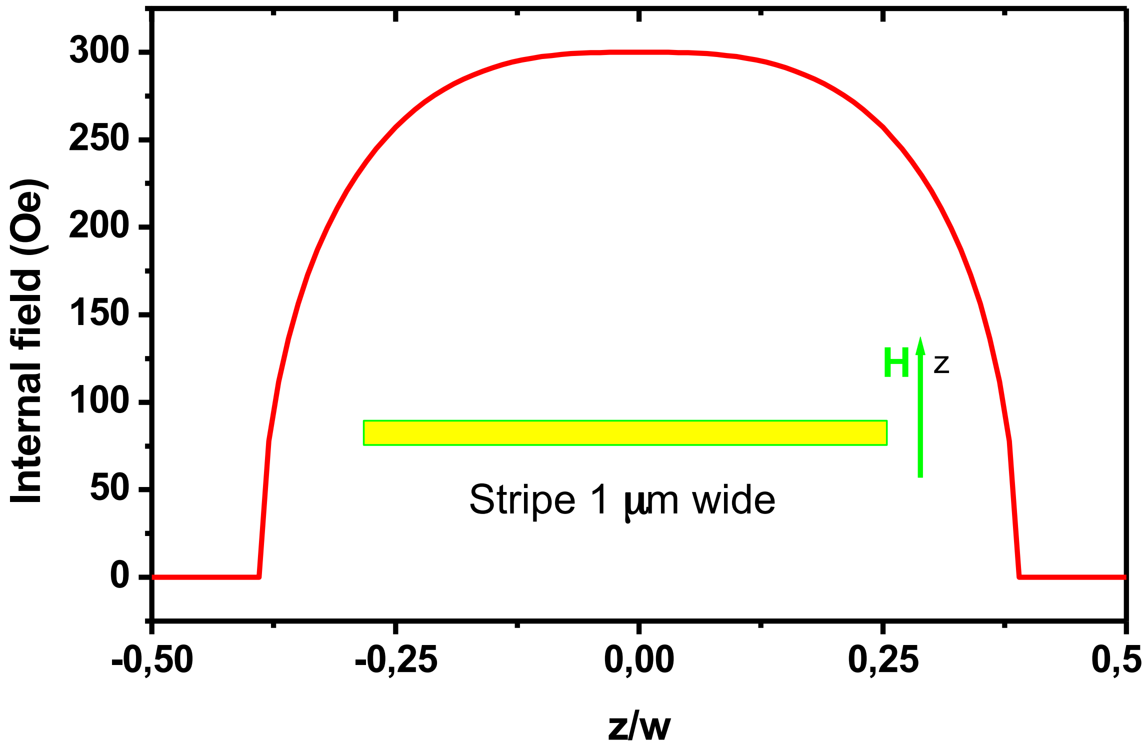


positions of the gaps



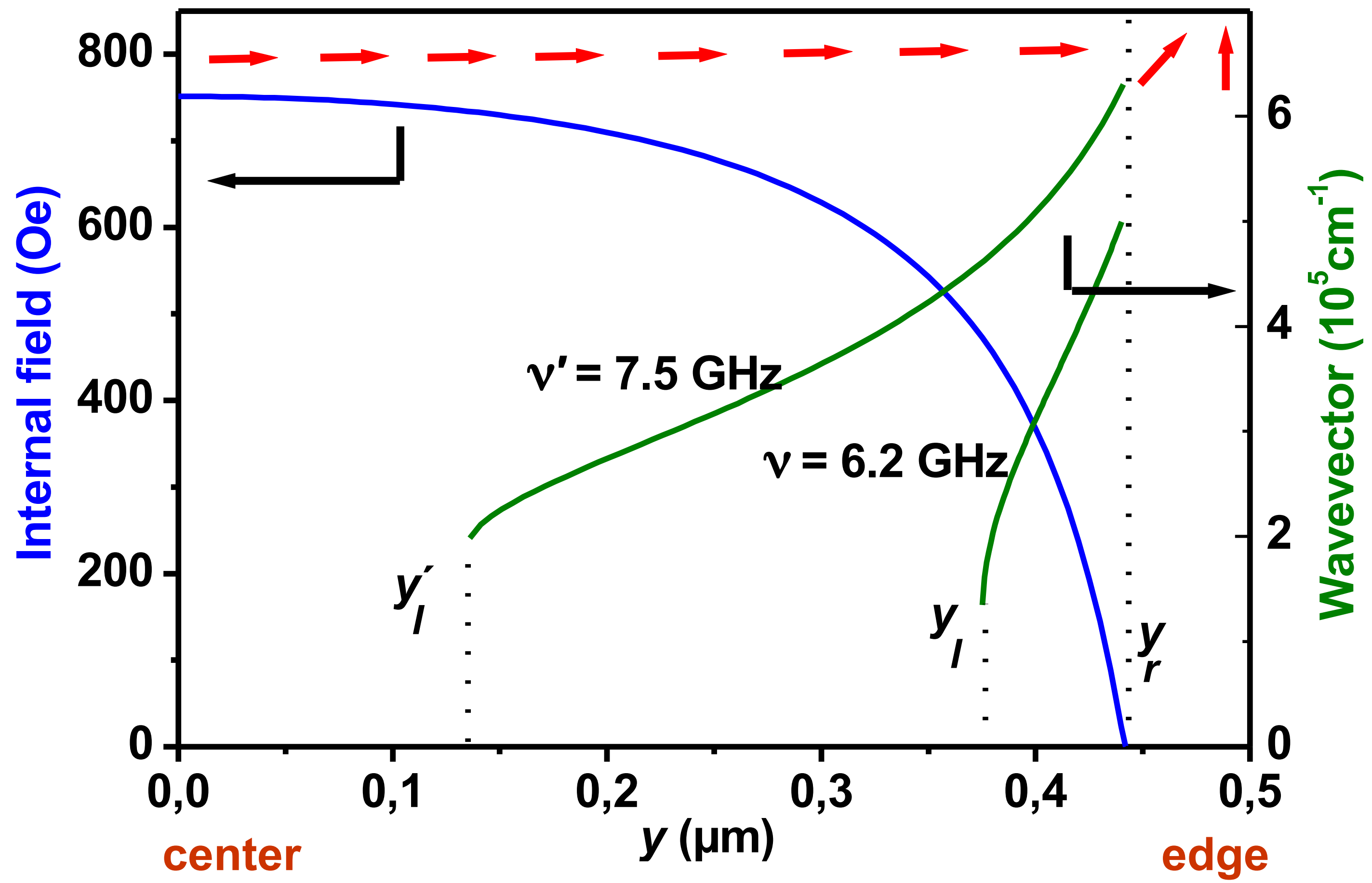
Application: Spin waves in films with internal field distribution





Regions with canted magnetization and zero internal field are located near the edges of the stripe

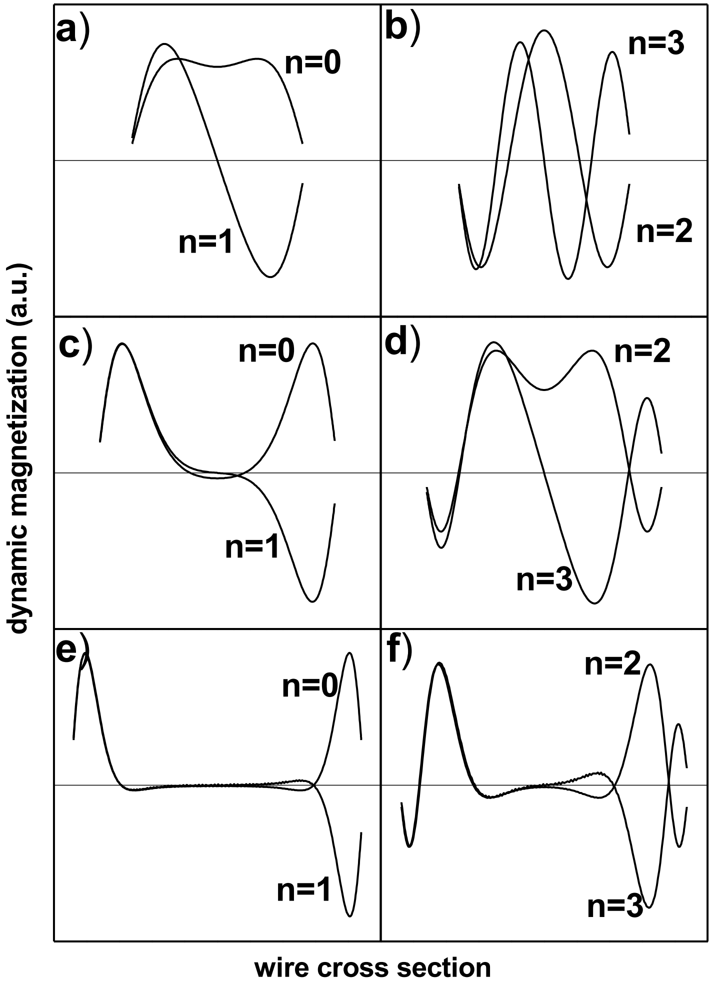
Dynamics in inhomogeneous stripe



y_l , y_l : turning points

y_r- y_l : localization length

Calculated mode profiles



300 Oe

450 Oe

950 Oe

Green's function approach using correct internal field distribution

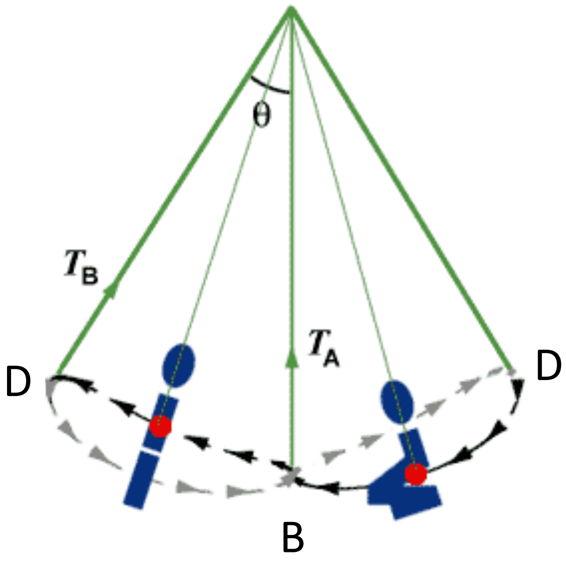
Numerical evaluation of the resulting integro-differential equations

Increasing localization with increasing internal field

Lecture 1 - CONTENT

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves

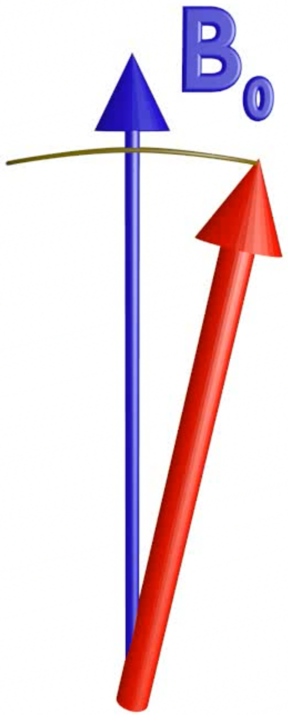
Parametric oscillator – a swing



www.hk-phy.org/articles/swing/swing_e.html

- Standing up at the point B:
  gain of energy: $E_1 = m \cdot g \cdot h$
- Kneeling at turning point D:
  loss of energy: $E_2 = m \cdot g \cdot h \cdot \cos(\phi)$
- Total gain of energy through changing of the position of
  the center of mass with doubled oscillation frequency: $E_1 - E_2 = m \cdot g \cdot h \cdot (1 - \cos(\phi))$

Analogy for the magnetic moment



- Variation of magnetic field with twice the frequency of the magnetic precession
- Interaction between longitudinal component of the dynamic magnetization and the varying magnetic field

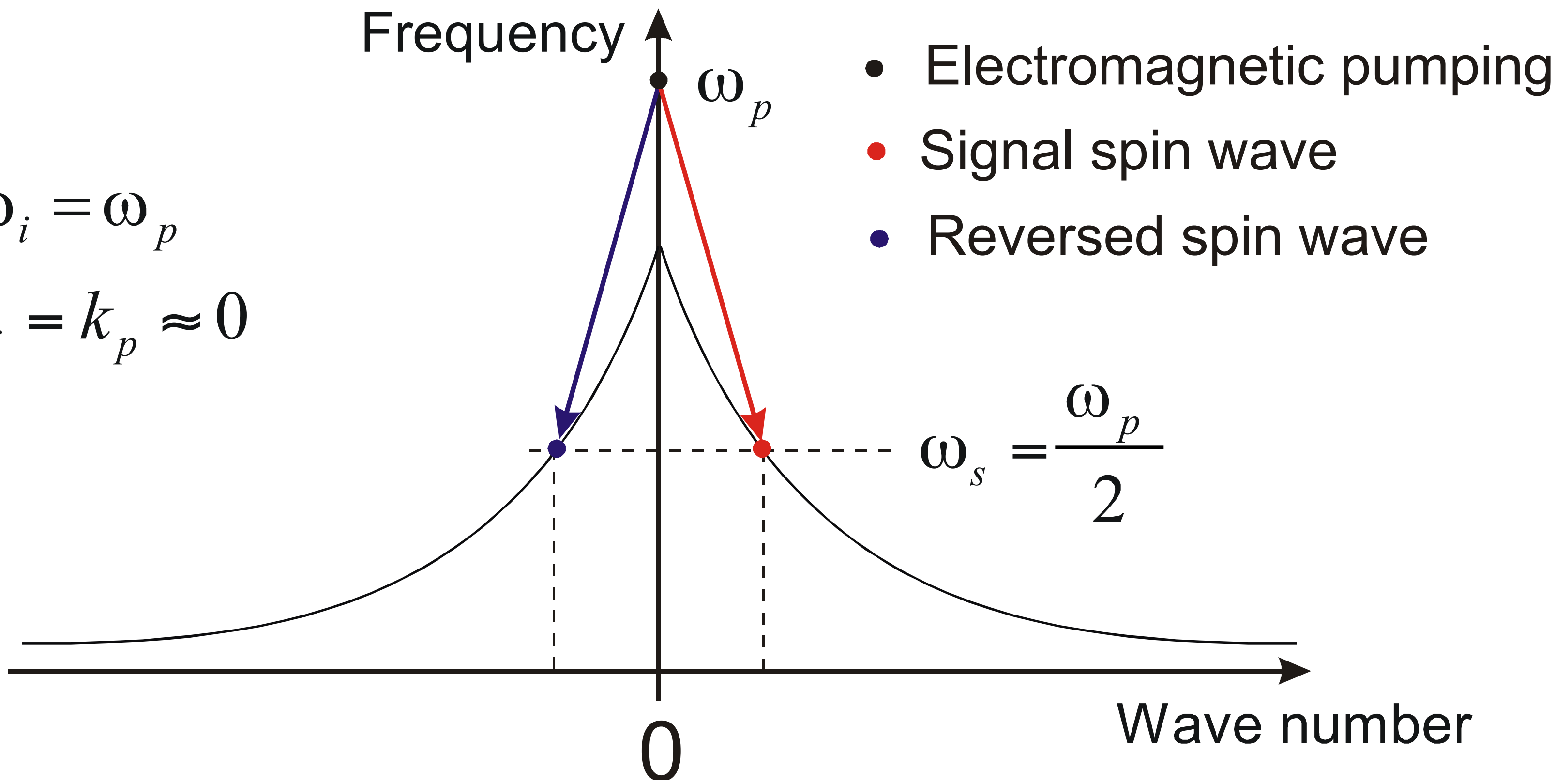
Animation: Dmytro Bozhko

Parametric amplification and phase conjugation

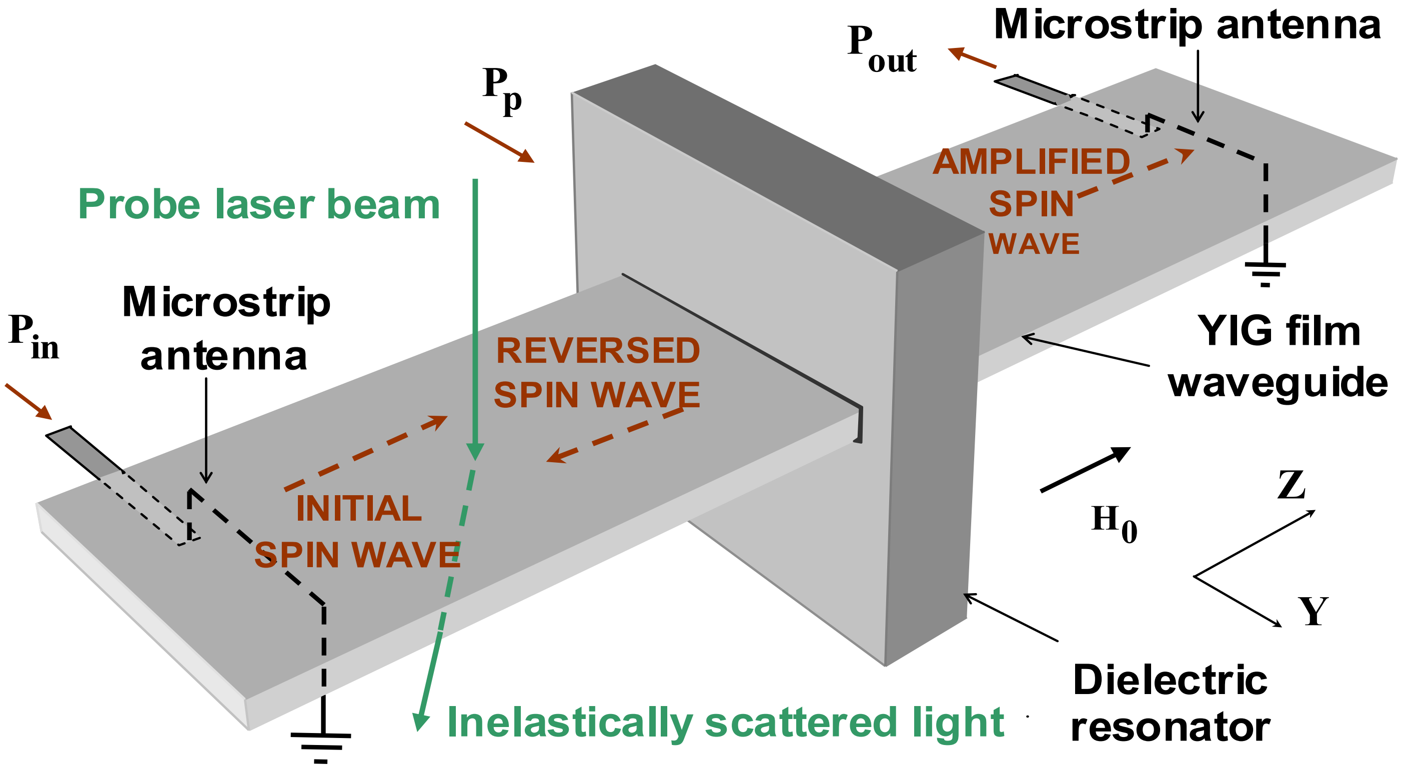
Splitting of a quasi-uniform electromagnetic pumping wave in two contra-propagating spin waves

$$\omega_s + \omega_i = \omega_p$$

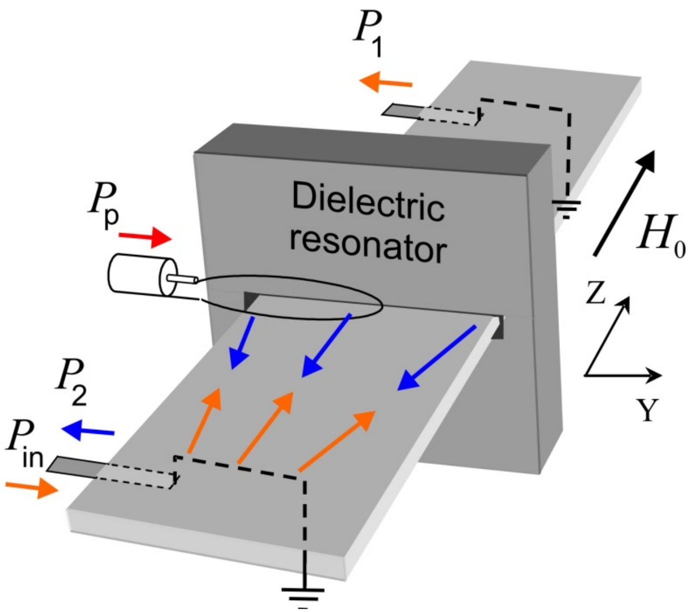
$$k_s + k_i = k_p \approx 0$$



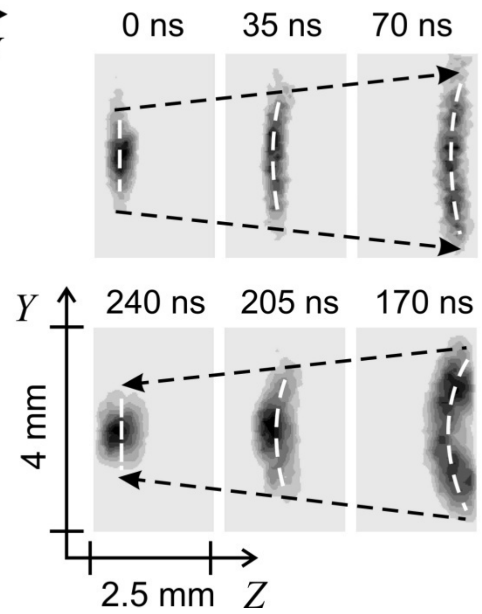
Bullet amplification (setup)



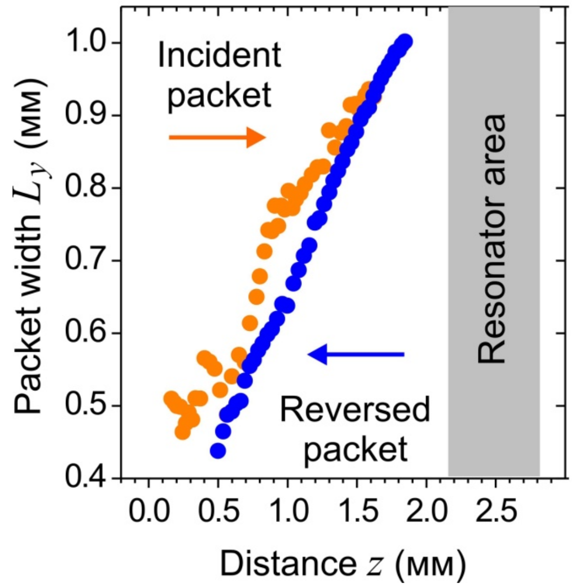
2D wave front reversal



wave front reversal observed in linear regime



Width of the linear BVMSW packet as a function of propagation distance



Input spin wave:

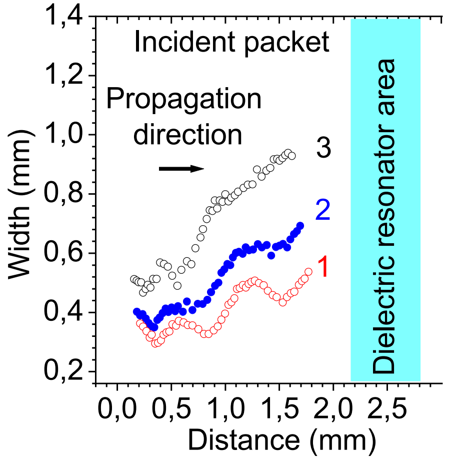
Bias magnetic field	$H_0 = 800$ Oe
Carrier frequency	$f_s = 3.966$ GHz
Carrier wave number	$k_s = 95$ rad/cm
Group velocity	$v_s = -2.0 \cdot 10^4$ m/s
Pulse duration	$\tau_s = 30$ ns
Pulse power	$P_{in} = 22$ mW

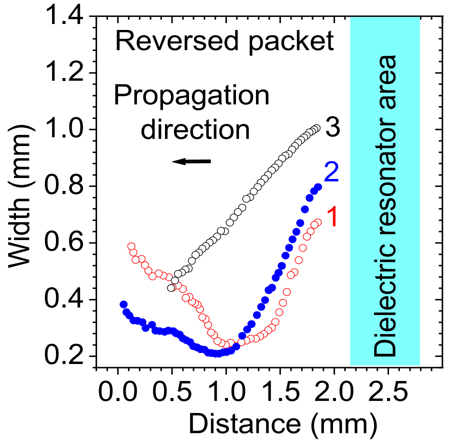
Pumping pulse:

Carrier frequency	$f_p = 2 f_s$
Power	$P_p = 30$ W
Duration	$\tau_p = 40$ ns

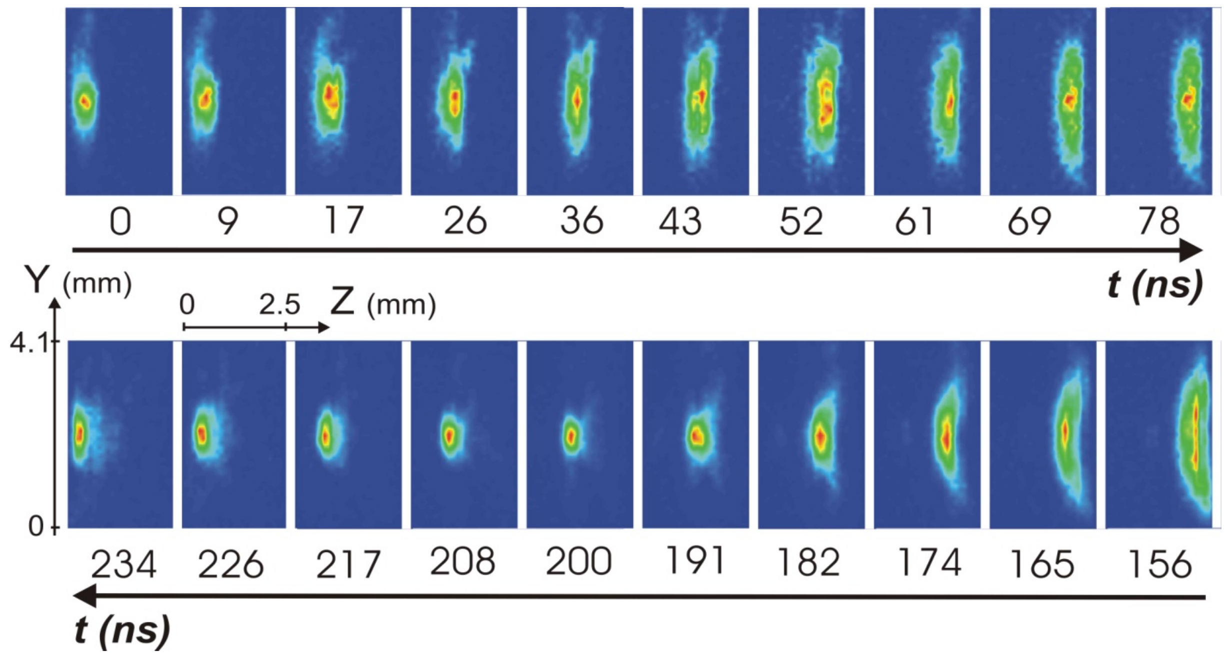
A.A. Serga *et al.*, Phys. Rev. Lett. **94**, 167202 (2005)

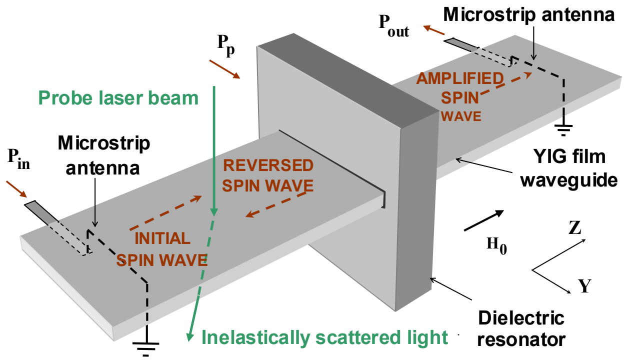
2D Wave Front Reversal and Bullet Formation





amplification up to nonlinear level





Lecture 1: What did we address so far:

- Basics: Spin Waves
- Experiment: Brillouin Light Scattering Spectroscopy
- Dynamics in Lateral Structures
- Spin Wave Tunneling Effect
- Parametric Generation and Amplification of Spin Waves